



# Implicit Function Ray Tracing

**Chris Birke**  
Secant Astronomy



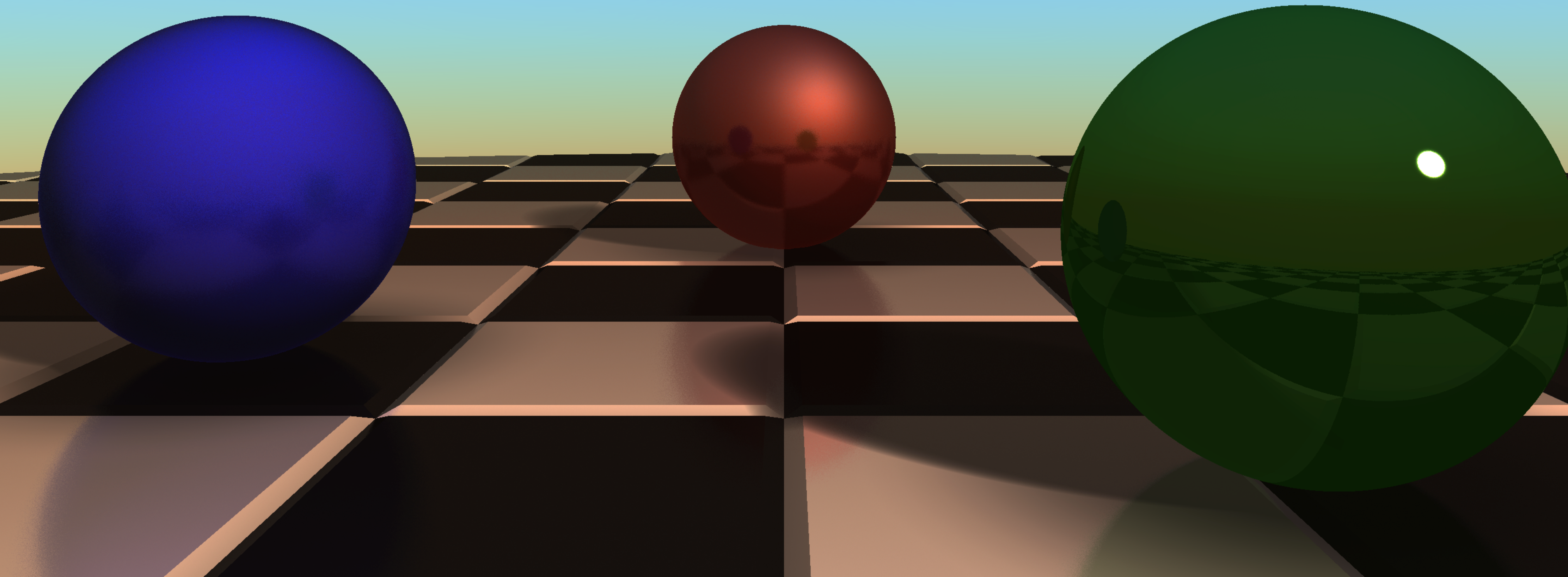
This talk is about rendering implicit functions, but it's still totally cool if you don't know math or code.

This talk is about art, too.





Ooo Shiny!





A few people to thank.





# Iñigo Quílez





# Iñigo Quílez

aka iq


$$v(t) = a + bt + ct^2$$

$$a = p = surf(a)$$



# Pablo Roman Andrioli





# Pablo Roman Andrioli

aka kali





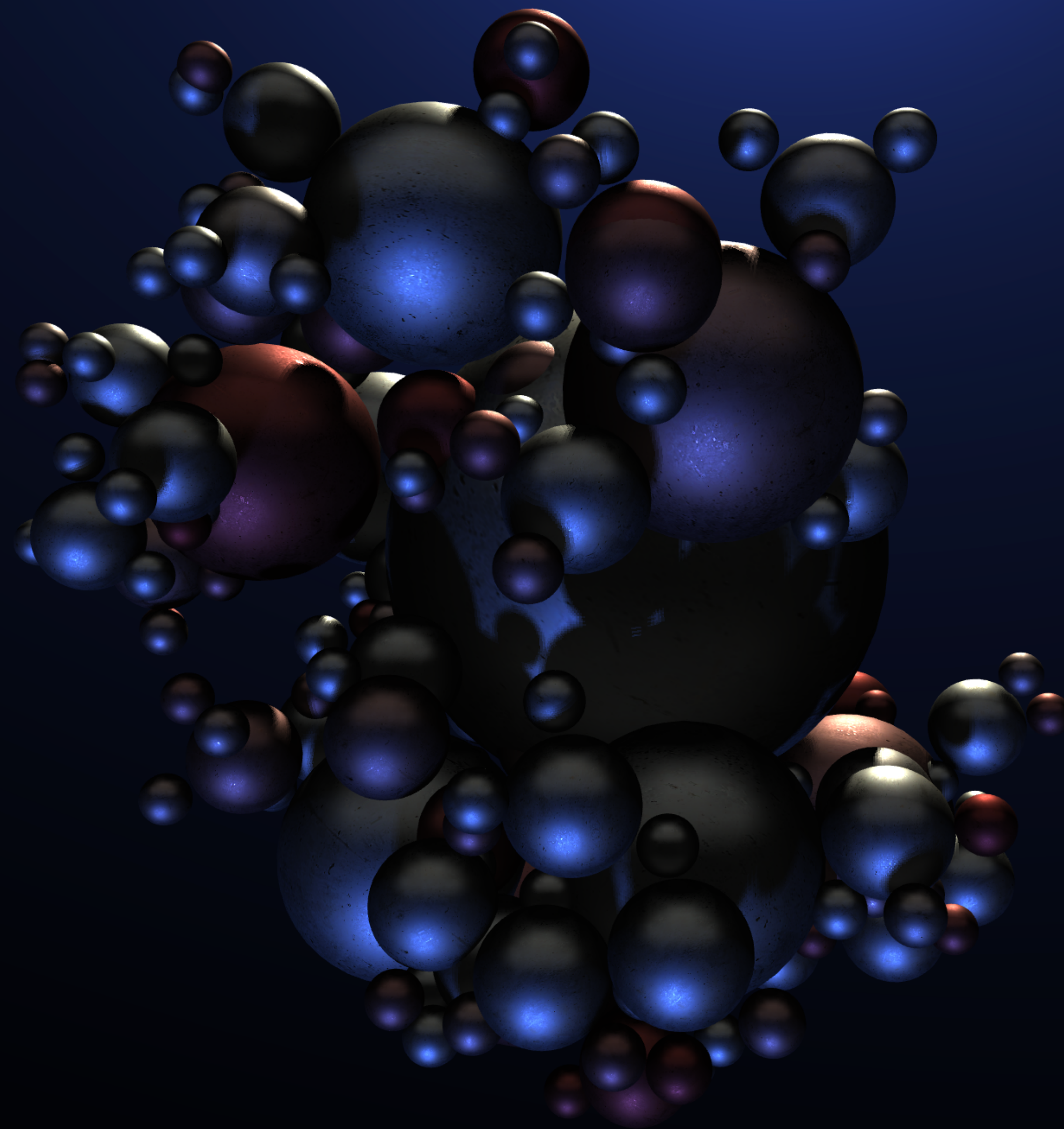
# Ricardo Cabello





# Ricardo Cabello

aka Mr.doob





A few places to thank.

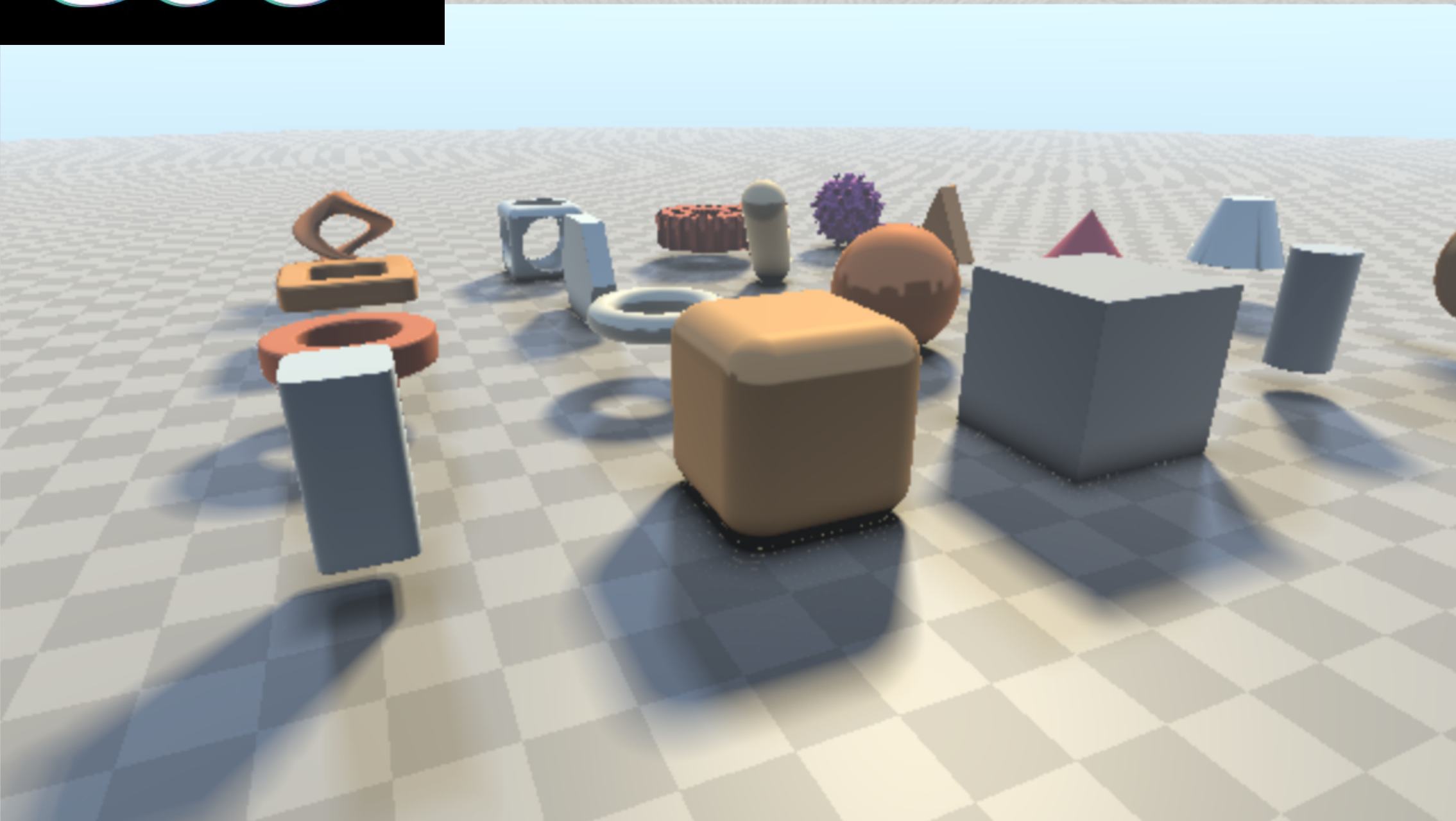




```
1 #ifdef GL_ES
2 precision mediump float;
3 #endif
4
5 #extension GL_OES_standard_derivatives : enable
6
7 uniform float time;
8 uniform vec2 mouse;
9 uniform vec2 resolution;
10
11 void main( void ) {
12
13     vec2 position = ( gl_FragCoord.xy / resolution.xy ) + mouse / 4.0;
14
15     float color = 0.0;
16     color += sin( position.x * cos( time / 15.0 ) * 80.0 ) + cos( position.y * cos( time / 15.0 ) * 10.0 );
17     color += sin( position.y * sin( time / 10.0 ) * 40.0 ) + cos( position.x * sin( time / 25.0 ) * 40.0 );
18     color += sin( position.x * sin( time / 5.0 ) * 10.0 ) + sin( position.y * sin( time / 35.0 ) * 80.0 );
19     color *= sin( time / 10.0 ) * 0.5;
20
21     gl_FragColor = vec4( vec3( color, color * 0.5, sin( color + time / 3.0 ) * 0.75 ), 1.0 );
22
23 }
```

# GLSL Sandbox





51.99 44.6 fps



## Raymarching - Primitives

58441 319 40

Uploaded by iq in 2013-Mar-25

Tags: procedural, 3d, raymarching, distancefields, primitives

A set of primitives and combination functions, for reference. More info here:  
<http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

### Comments

Sign in to post a comment.

**hackthechad**, 2016-Feb-21

Awesome reference IQ--learning a lot! Quick question though. What are the bac and dom variables in your lighting calculations?

```
float bac = clamp( dot( nor, normalize(vec3(-lig.x,0.0,-lig.z))), 0.0, 1.0 ) * clamp( 1.0-pos.y,0.0,1.0 );  
float dom = smoothstep( -0.1, 0.1, ref.y );
```

**Doublefresh**, 2016-Feb-13

What does the 'c' parameter in sdCone do?

+ Image

### Shader Inputs

```
1 // Created by inigo quilez - iq/2013  
2 // License Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License.  
3  
4 // A list of usefull distance function to simple primitives, and an example on how to  
5 // do some interesting boolean operations, repetition and displacement.  
6 //  
7 // More info here: http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm  
8  
9 float sdPlane( vec3 p )  
10 {  
11     return p.y;  
12 }  
13  
14 float sdSphere( vec3 p, float s )  
15 {  
16     return length(p)-s;  
17 }  
18  
19 float sdBox( vec3 p, vec3 b )  
20 {  
21     vec3 d = abs(p) - b;  
22     return min(max(d.x,max(d.y,d.z)),0.0) + length(max(d,0.0));  
23 }  
24  
25 float sdEllipsoid( in vec3 p, in vec3 r )  
26 {  
27     return (length( p/r ) - 1.0) * min(min(r.x,r.y),r.z);  
28 }  
29  
30 float udRoundBox( vec3 p, vec3 b, float r )  
31 {  
32     return length(max(abs(p)-b,0.0))-r;  
33 }  
34  
35 float sdTorus( vec3 p, vec2 t )  
36 {  
37     return length( vec2(length(p.xz)-t.x,p.y) )-t.y;  
38 }  
39
```

6757 chars



Shadertoy

iChannel0

iChannel1

iChannel2

iChannel3



# OPENING



# Techno / Chiptunes



# Many Cubes

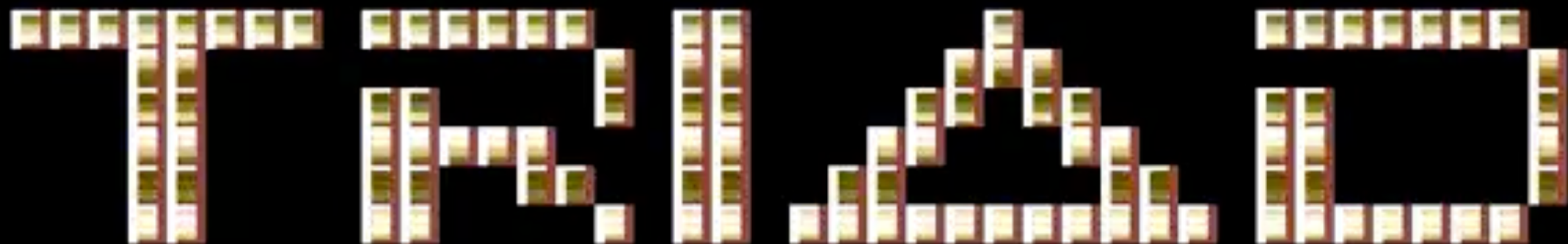


# Procedural Graphics

The butterfly effect - Andromeda Software Development



*DEALER QUALITY SOFTWARE*



APRIL, MAY, JUNE, JULY





FAIRLIGHT PRESENTS  
FULL CONTACT FROM TEAM 17

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# AAZOO 19XX



REZ



Greetings! Ubisoft • Nice one, bring 'em on!

WARNING! No permanent internet connection needed



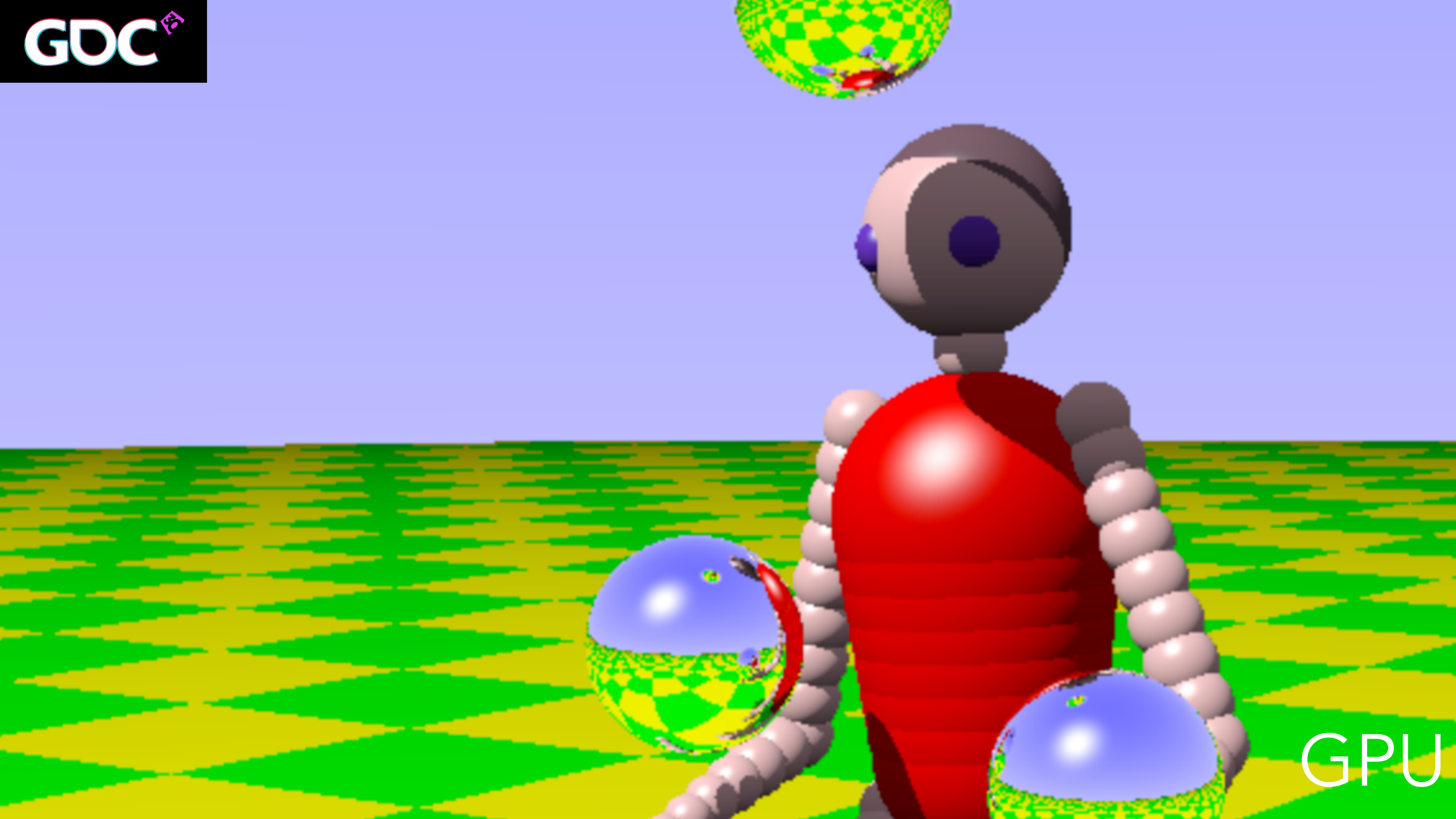






Amiga



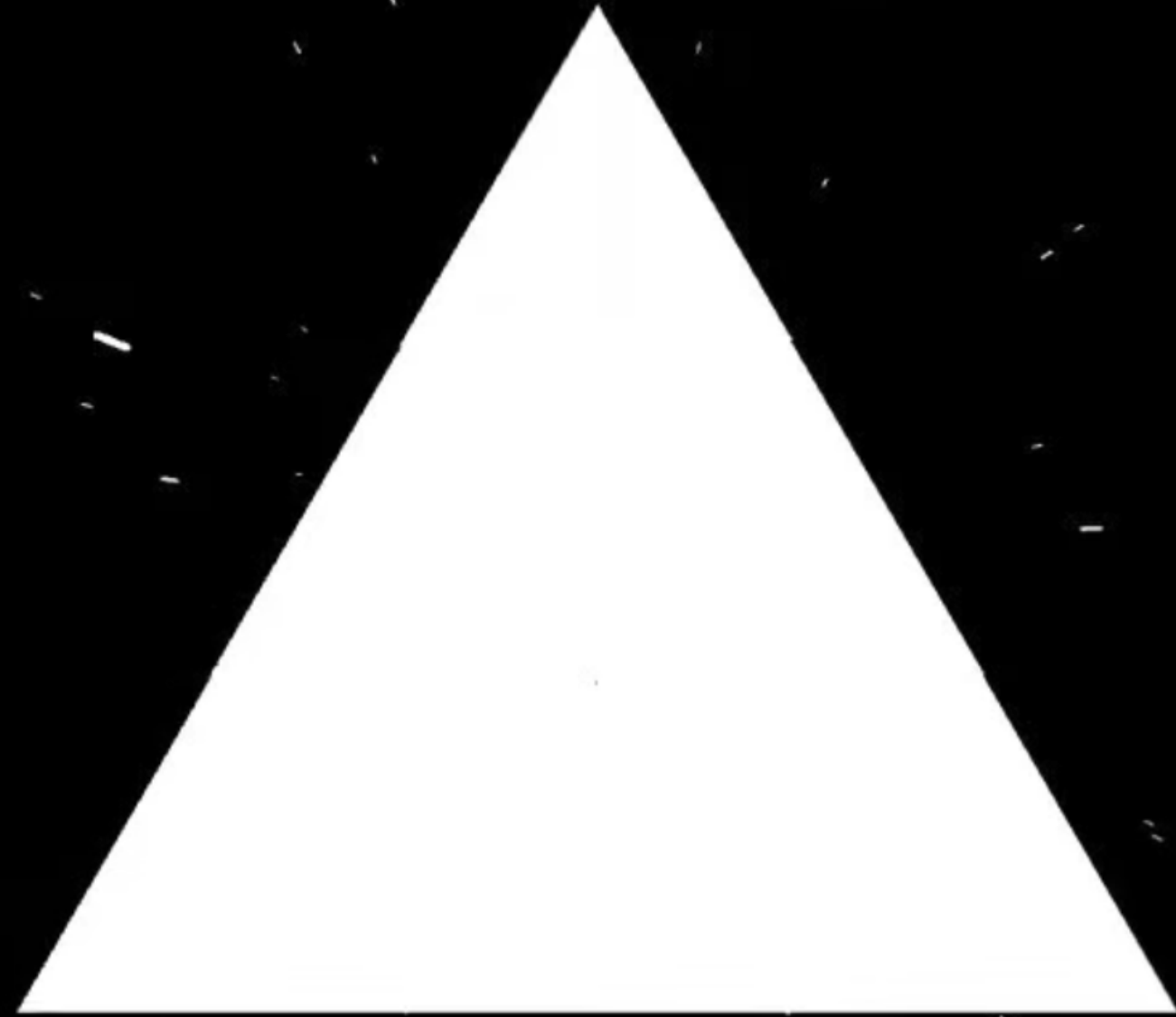


GPU















# Demoscene

Many Cubes

Much Techno

C64/Amiga/GPU

Procedural

Still Relevant

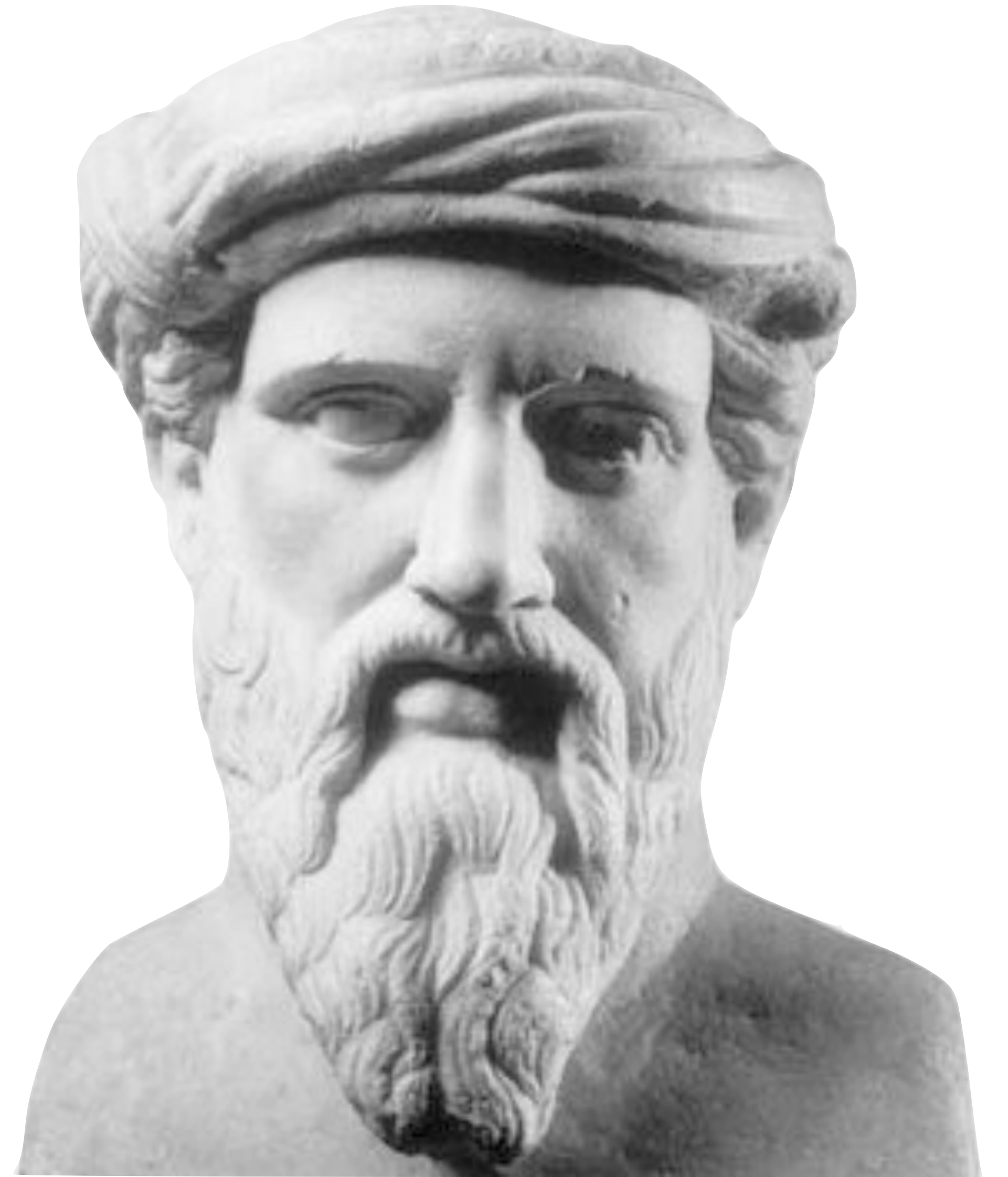


SF demoscene  
info goes here....





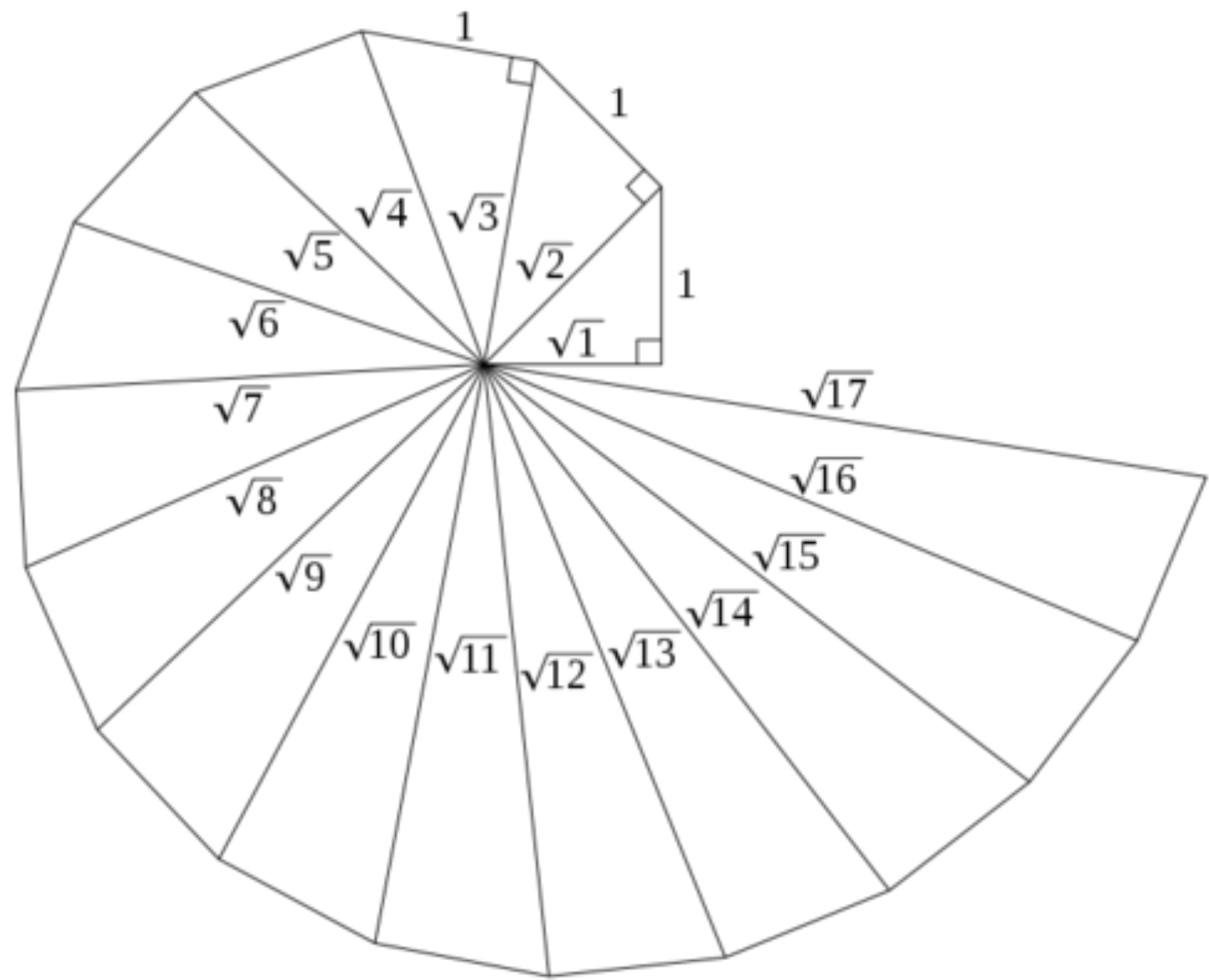
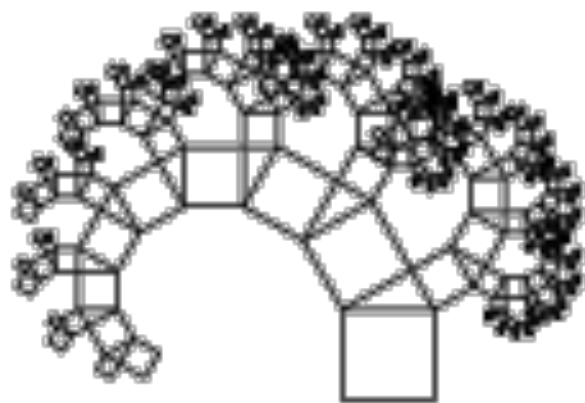
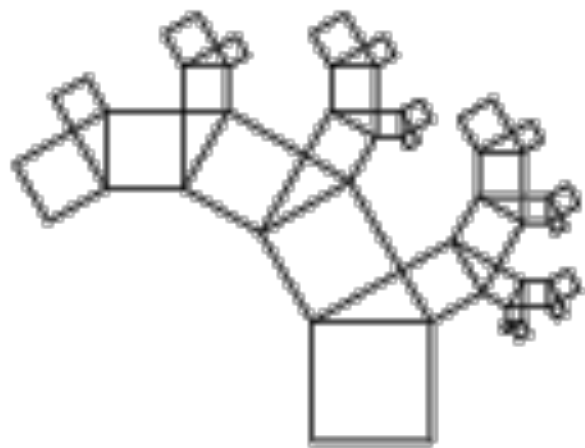
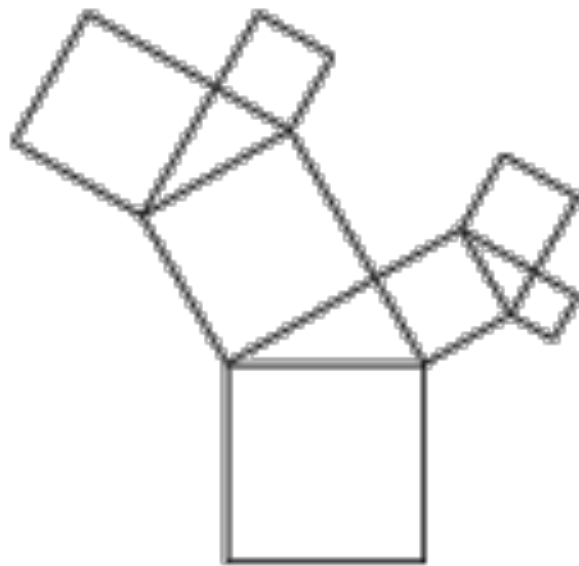
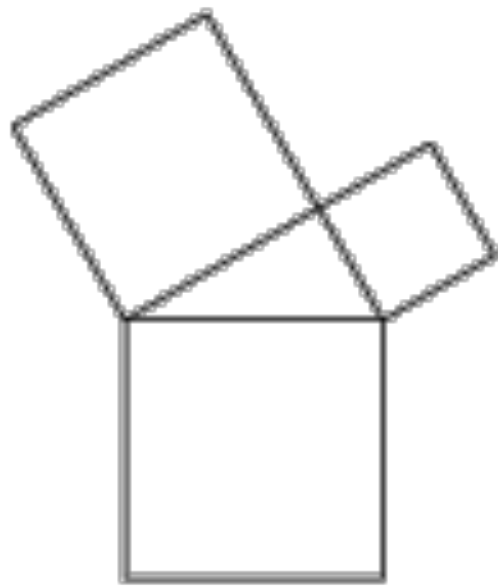
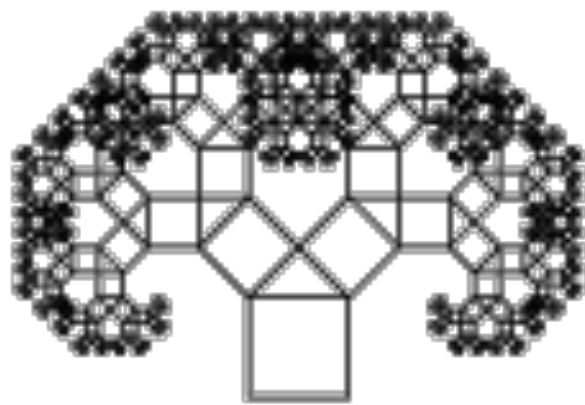
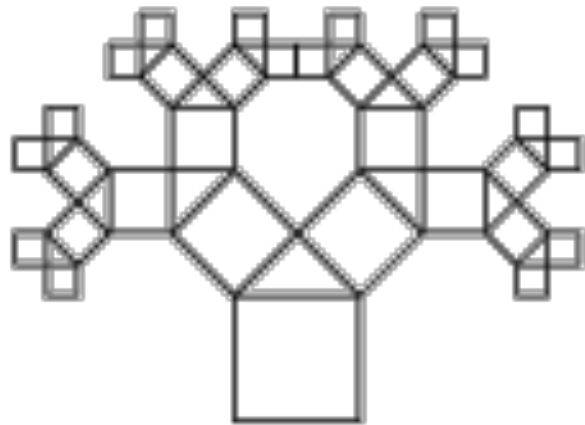
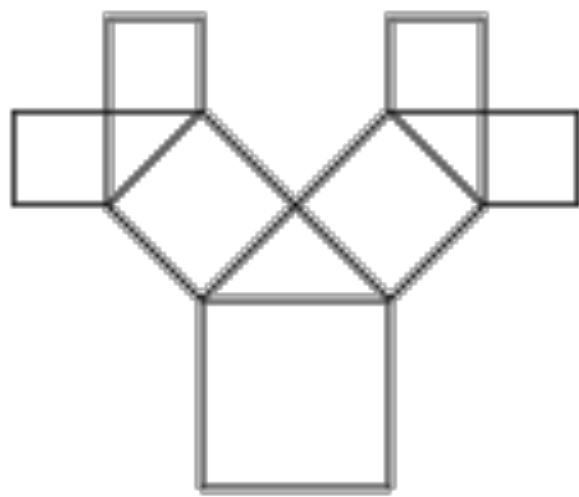
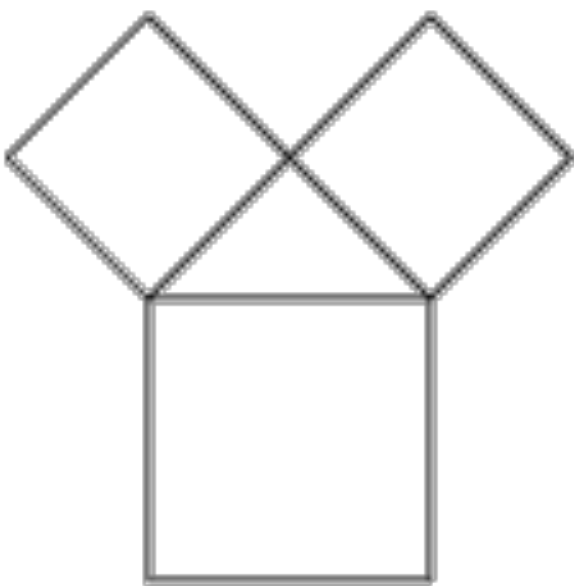
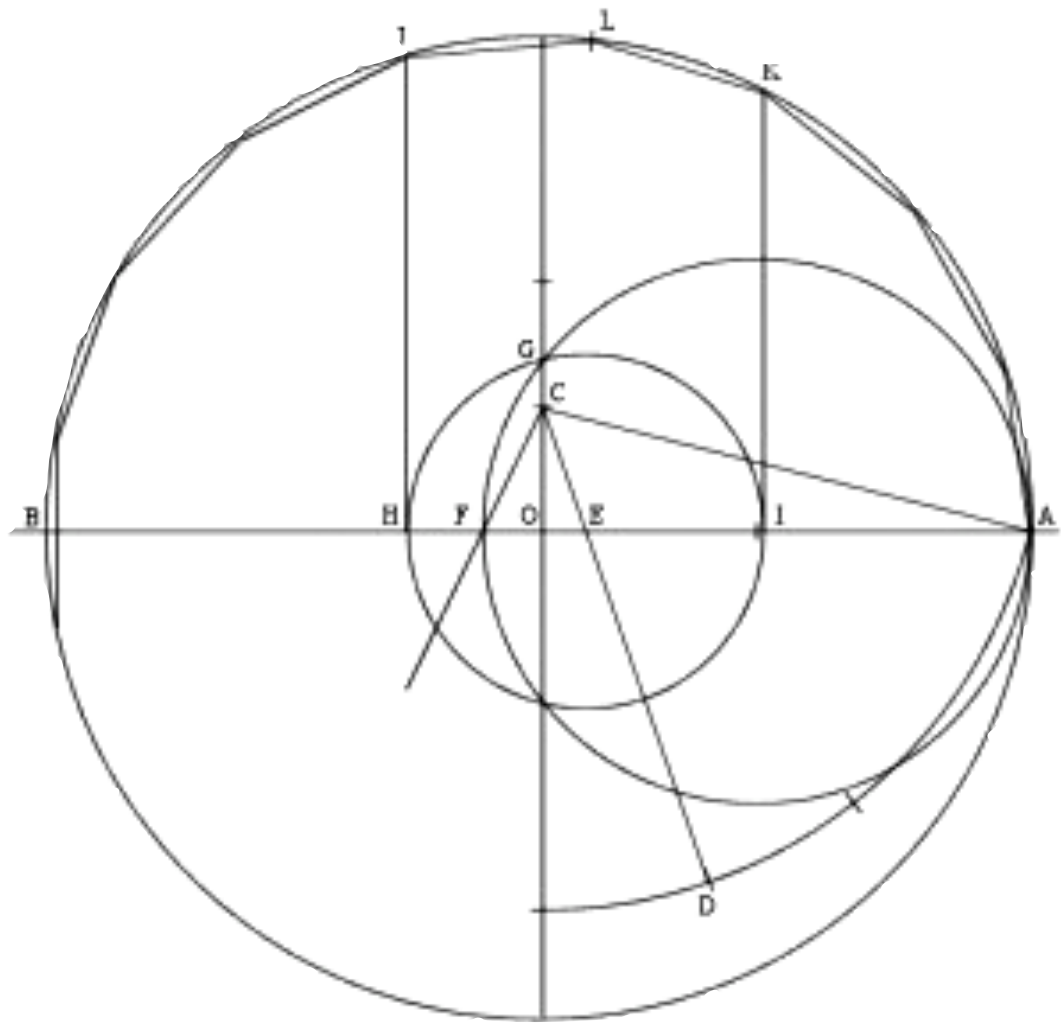
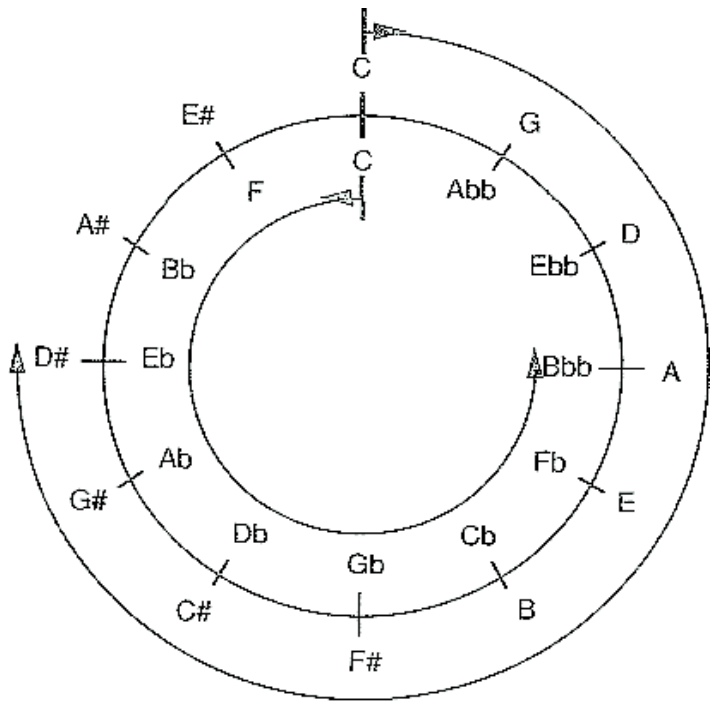
# Πυθαγόρας





# Pythagoras

Discovered tons of crazy stuff.







2500 years ago.



# Pythagoras

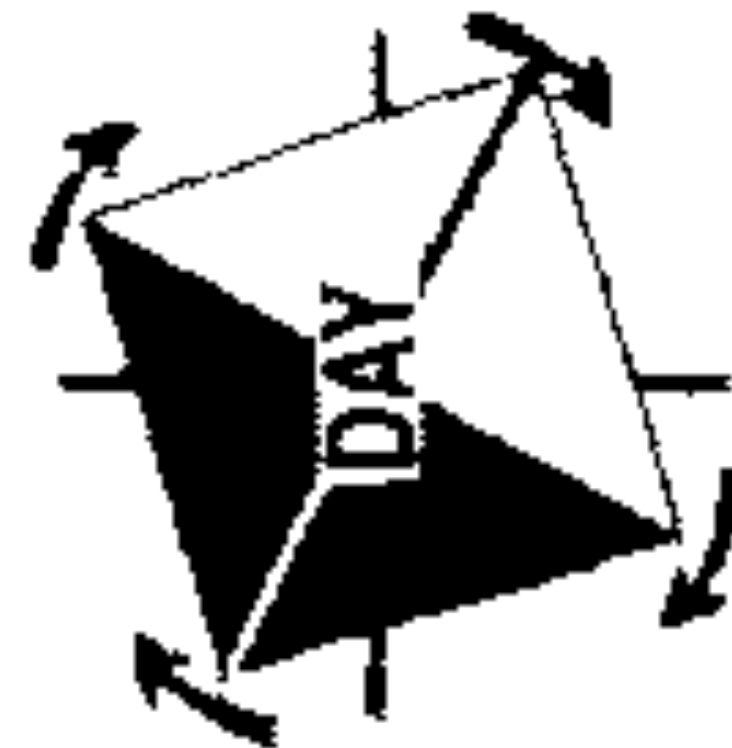
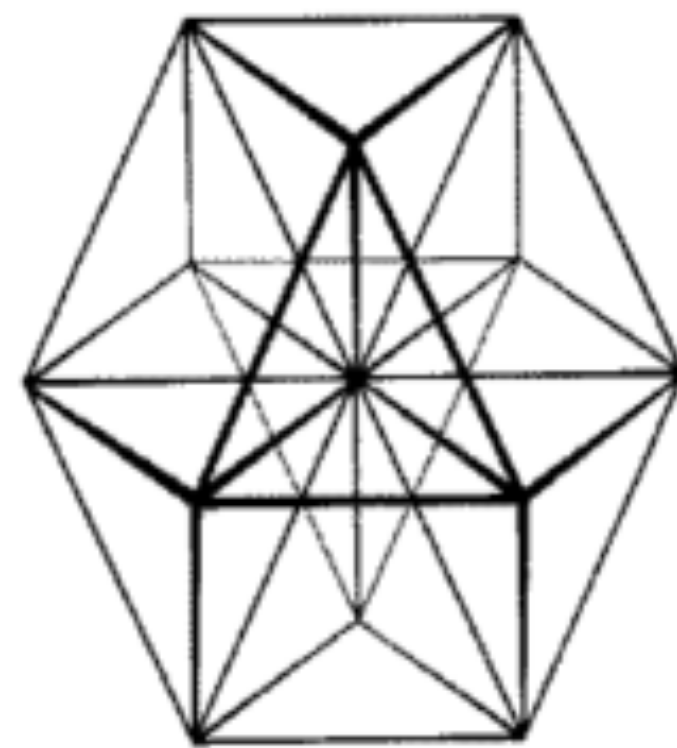
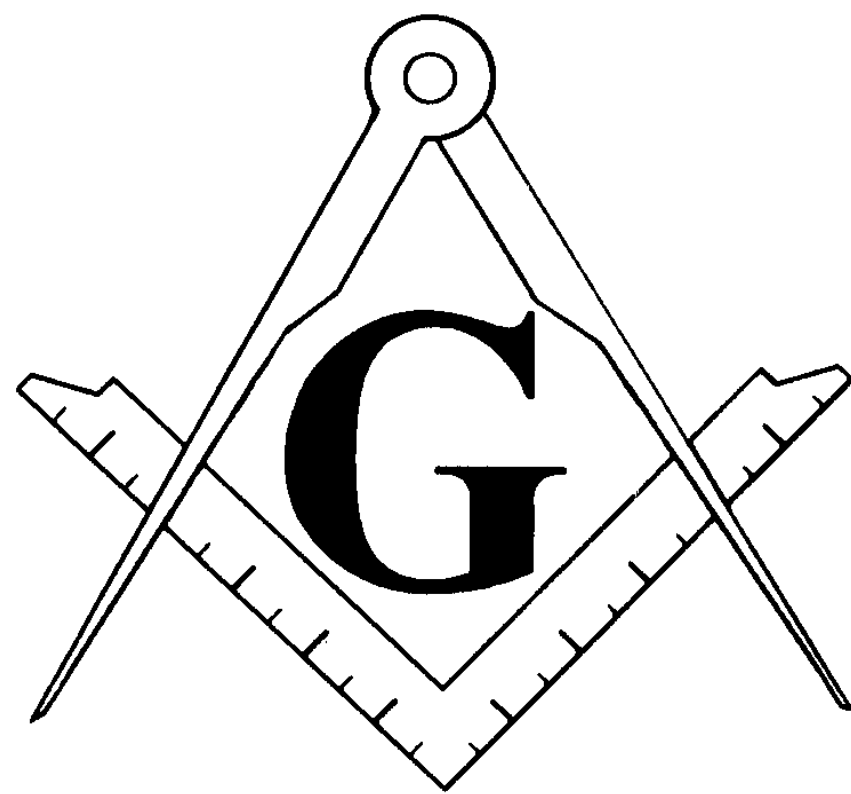
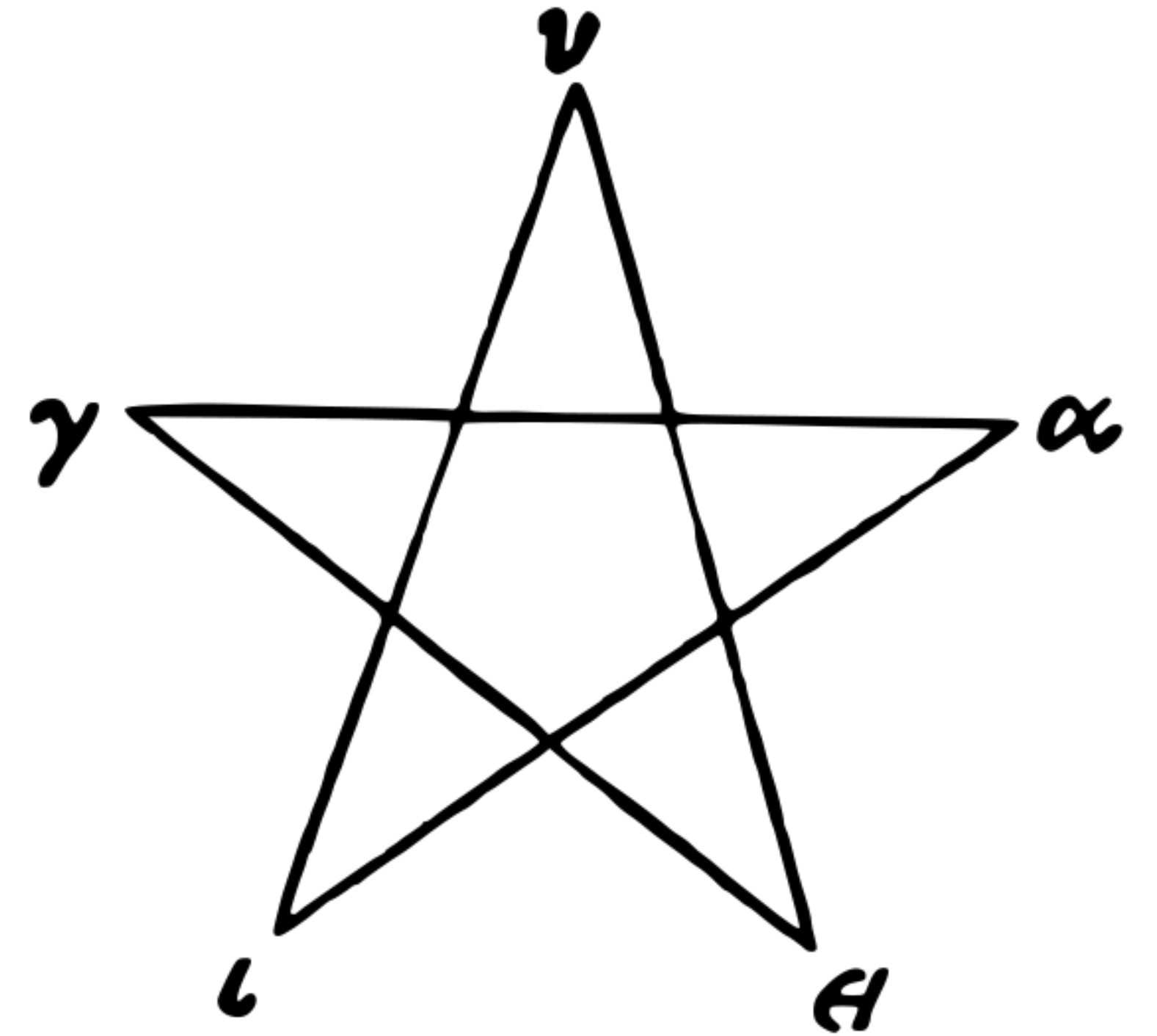
Straight edge (and compass.)





# Pythagoras

Spawned numerous math cults.

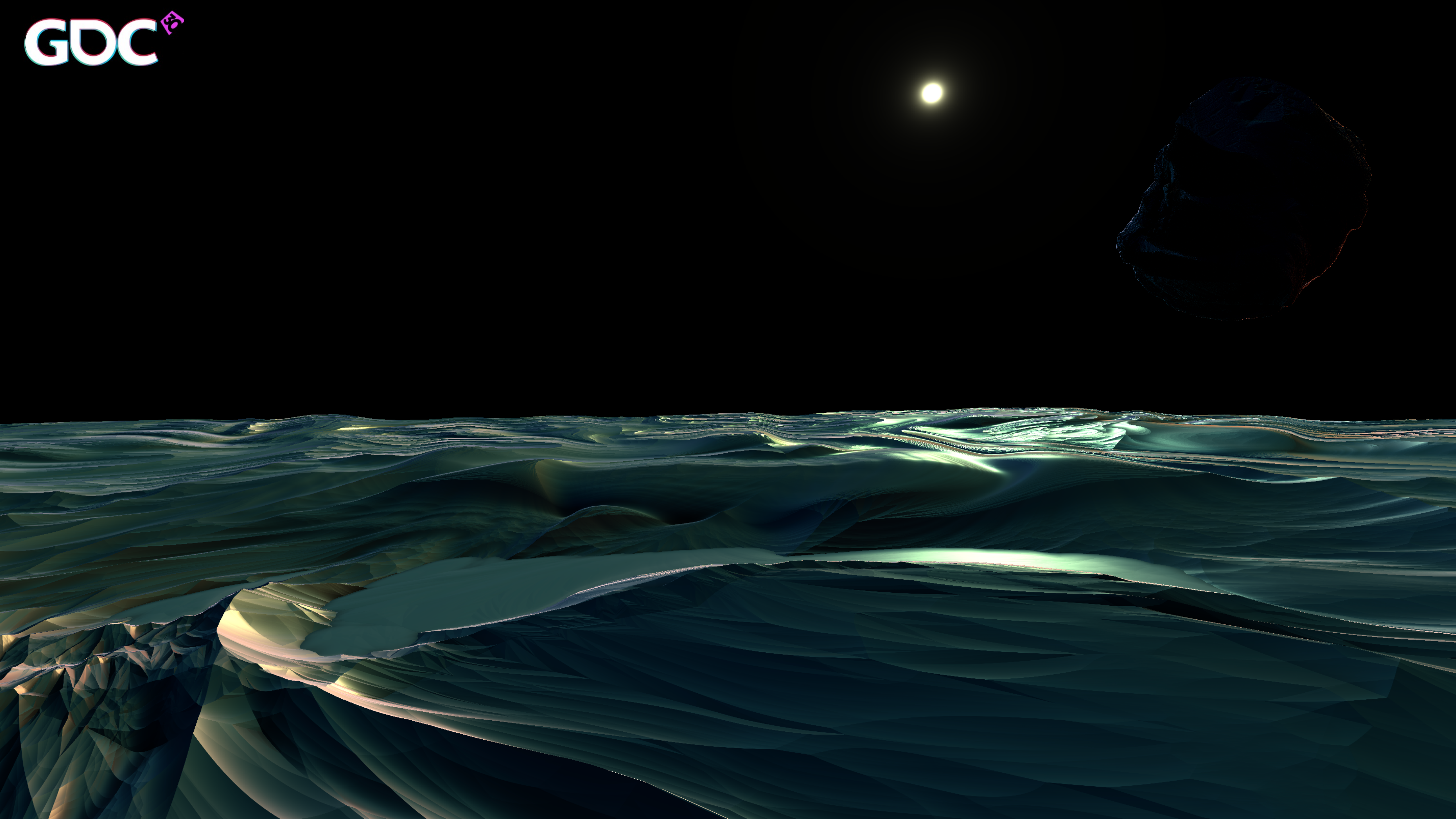




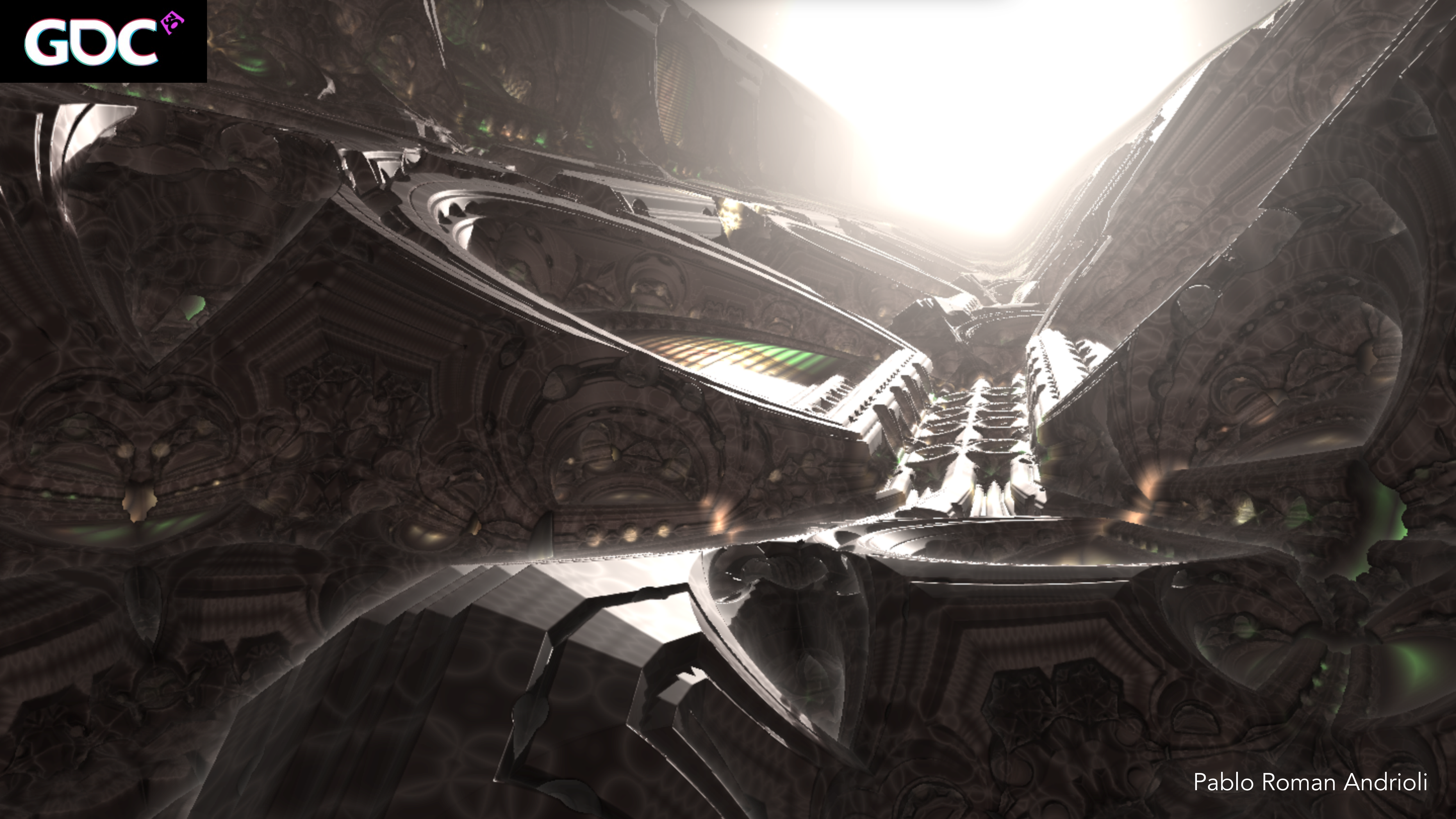


The universe is math.

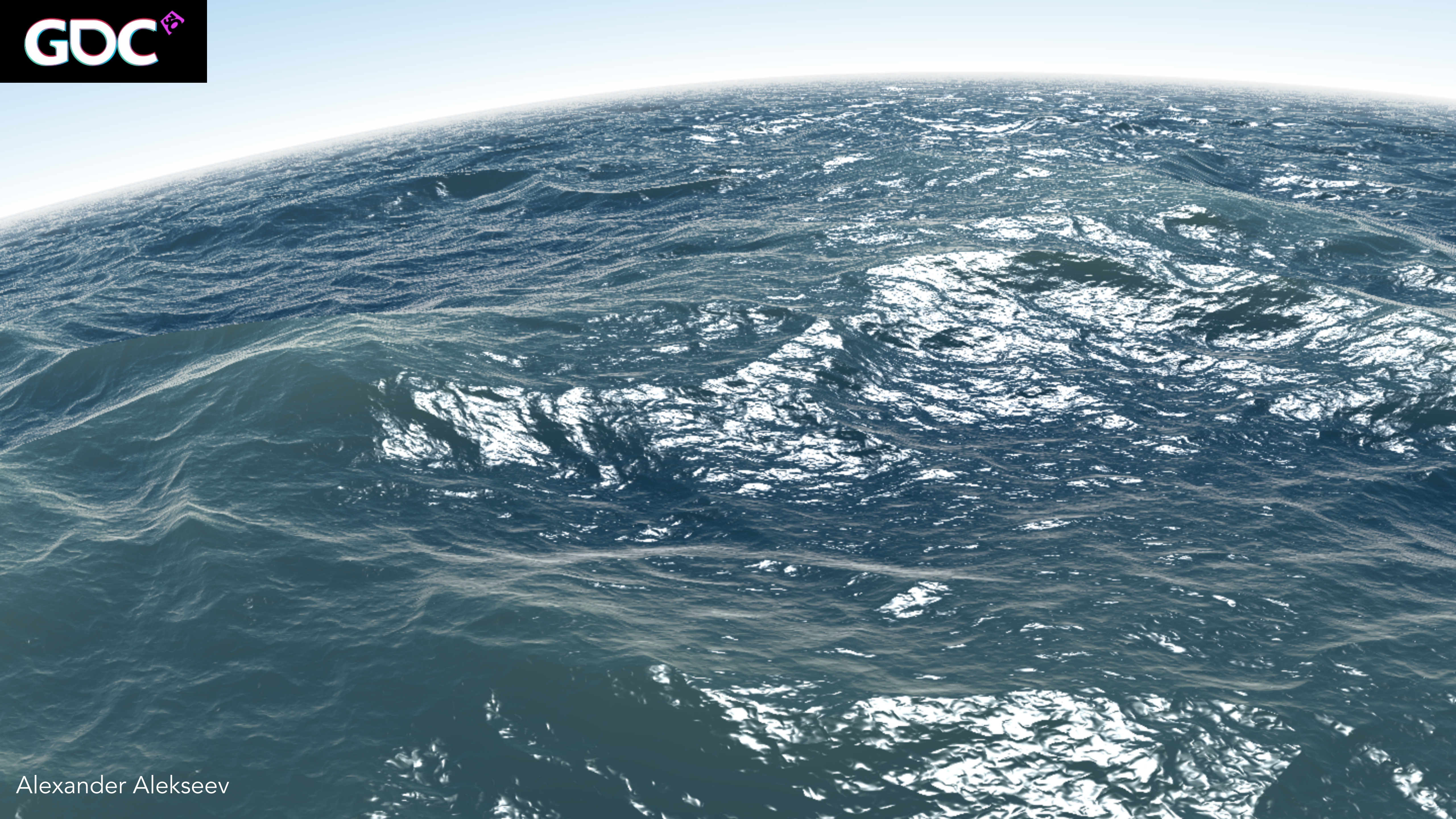




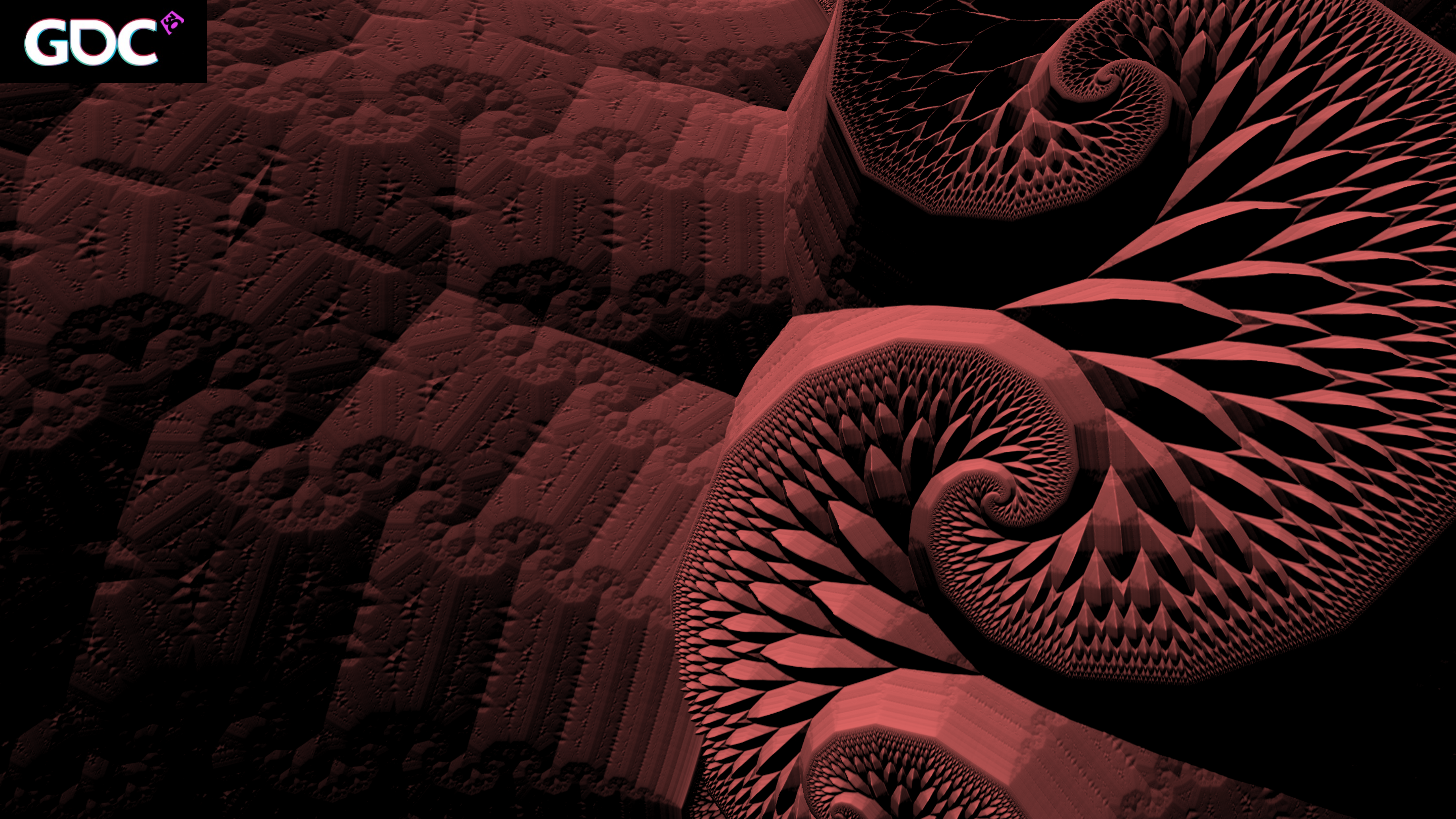
















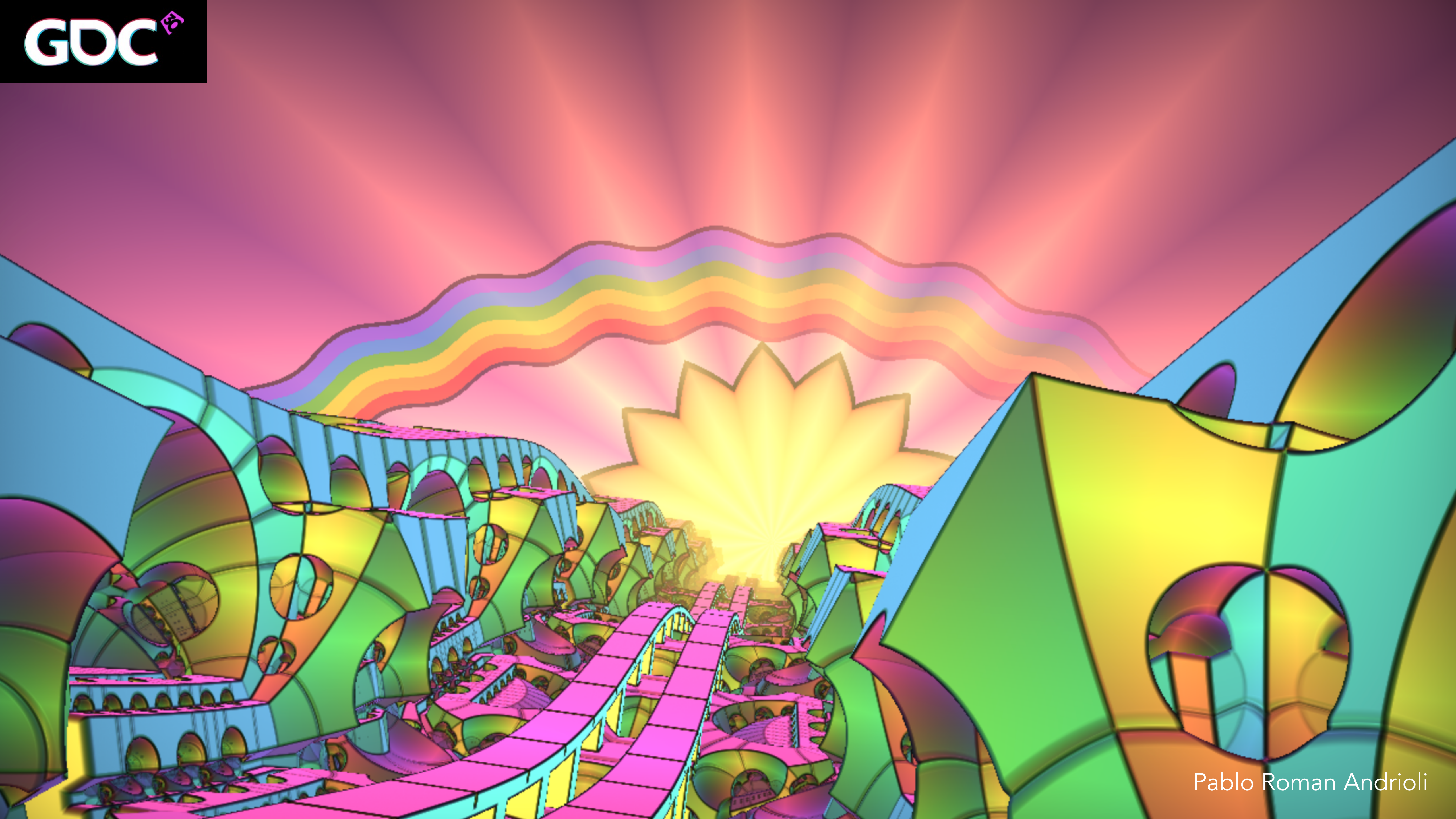




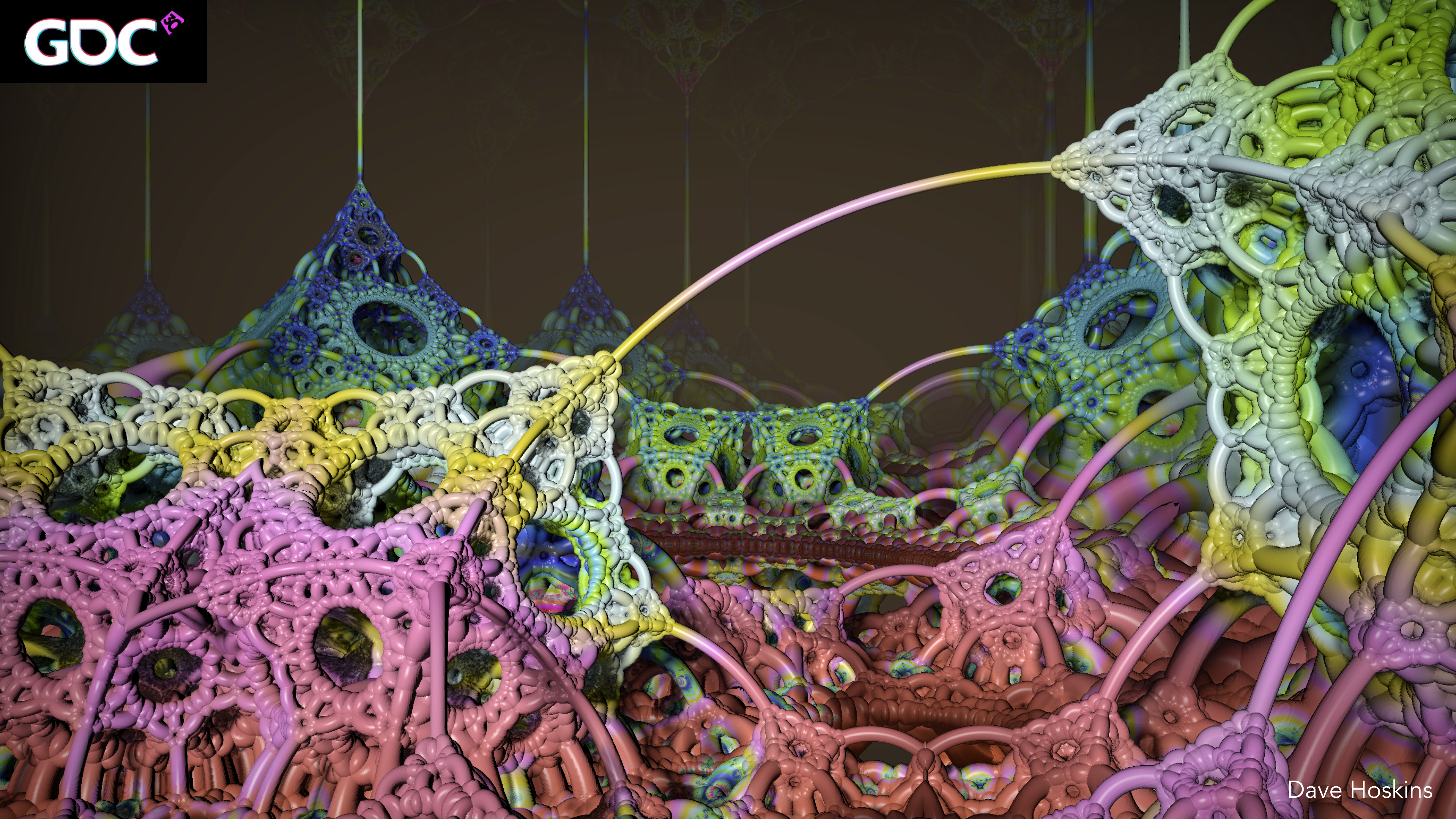




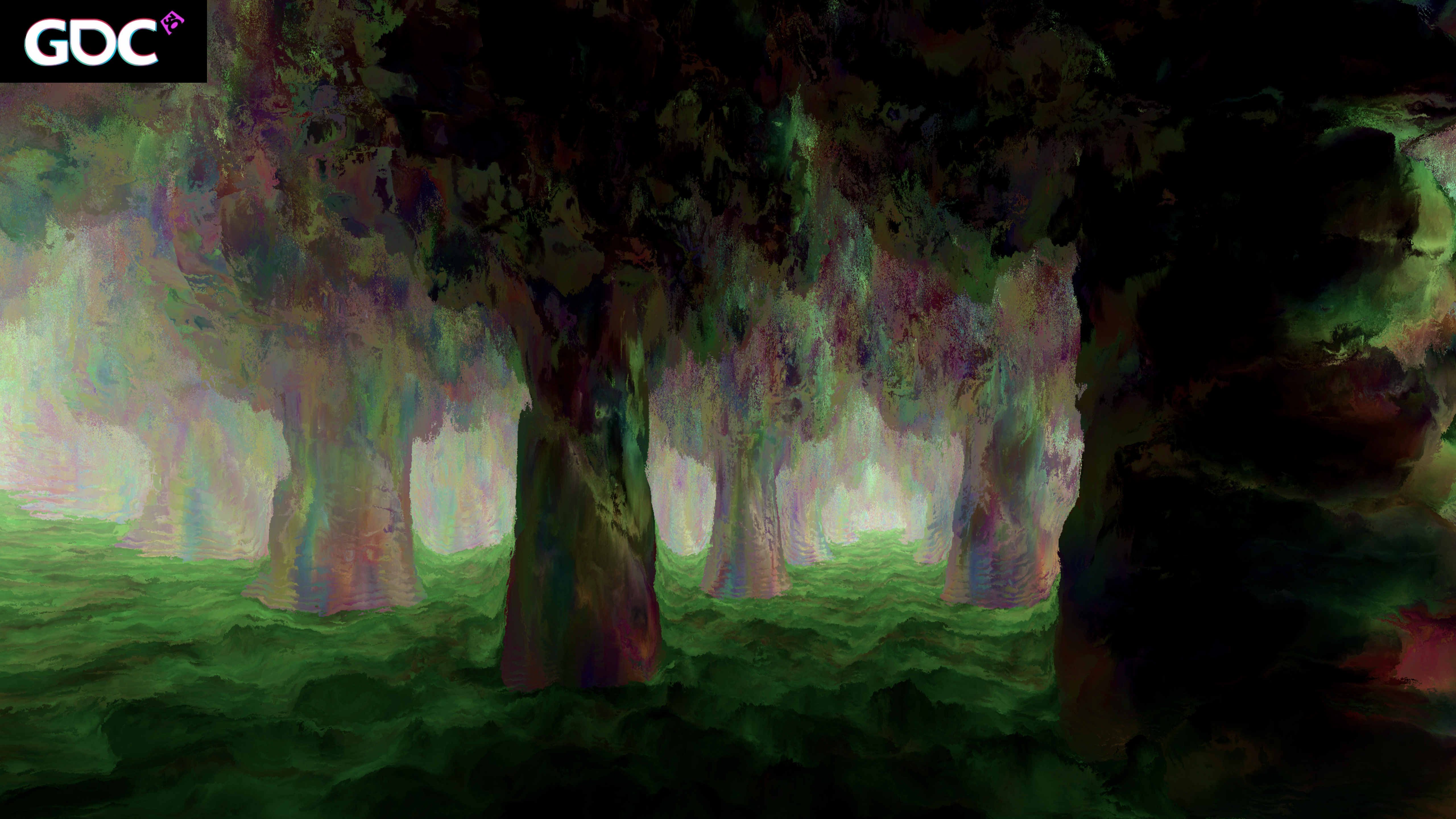




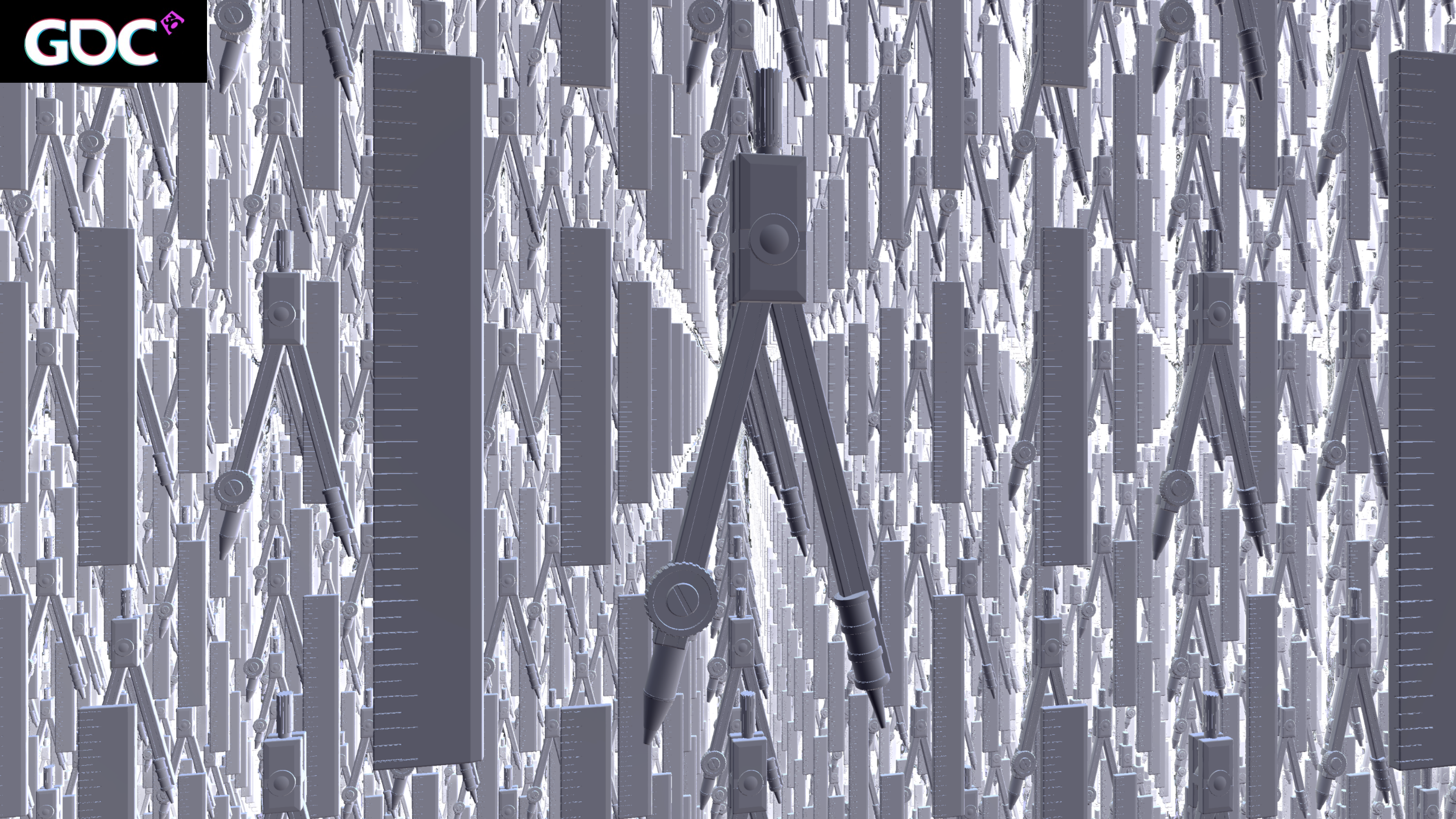
















If the universe is math...



Raytracing is a nice way to visit.





.





.

This is the point of the talk.





Actually, there are two points.

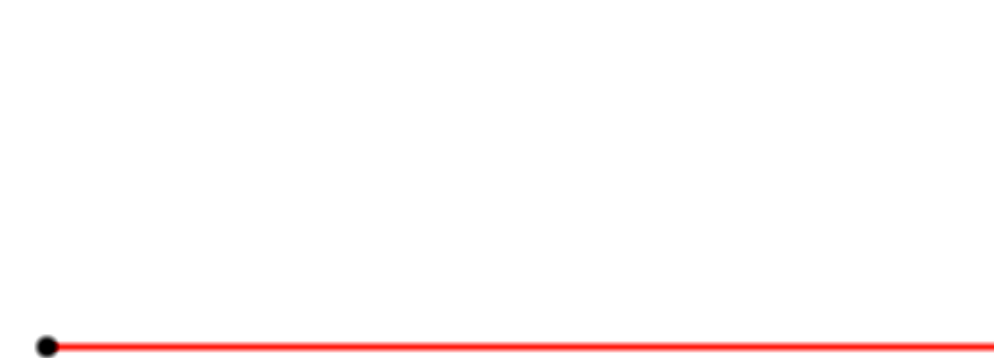


.

.

How far apart are they?





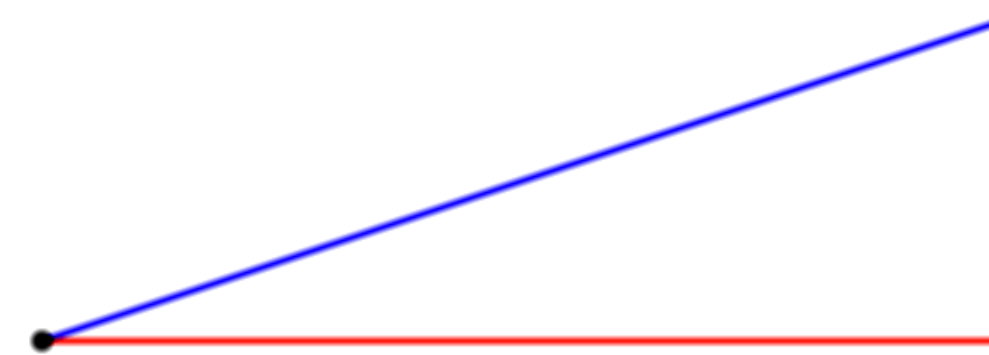
Time for math.





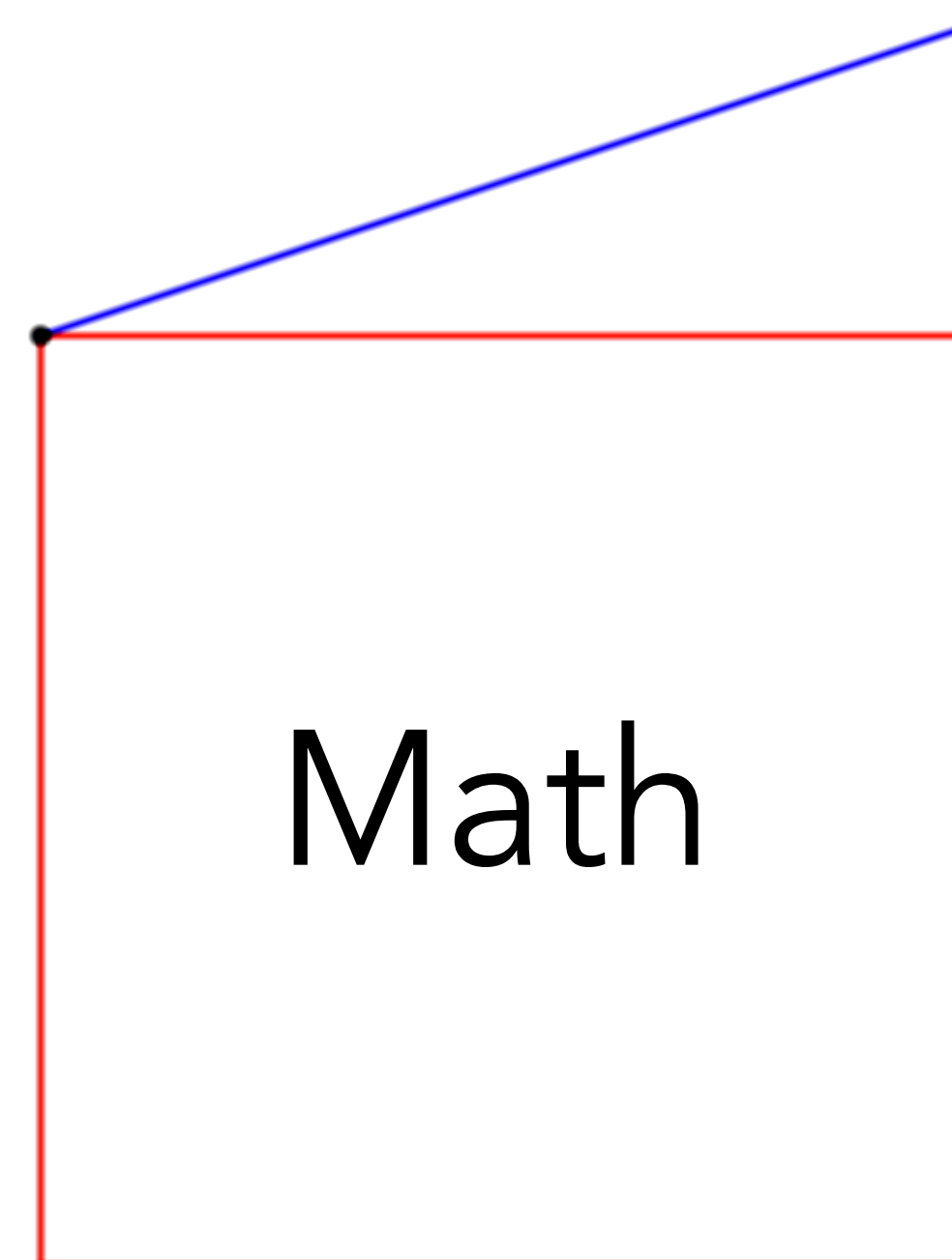
But it's hard to measure math against art.



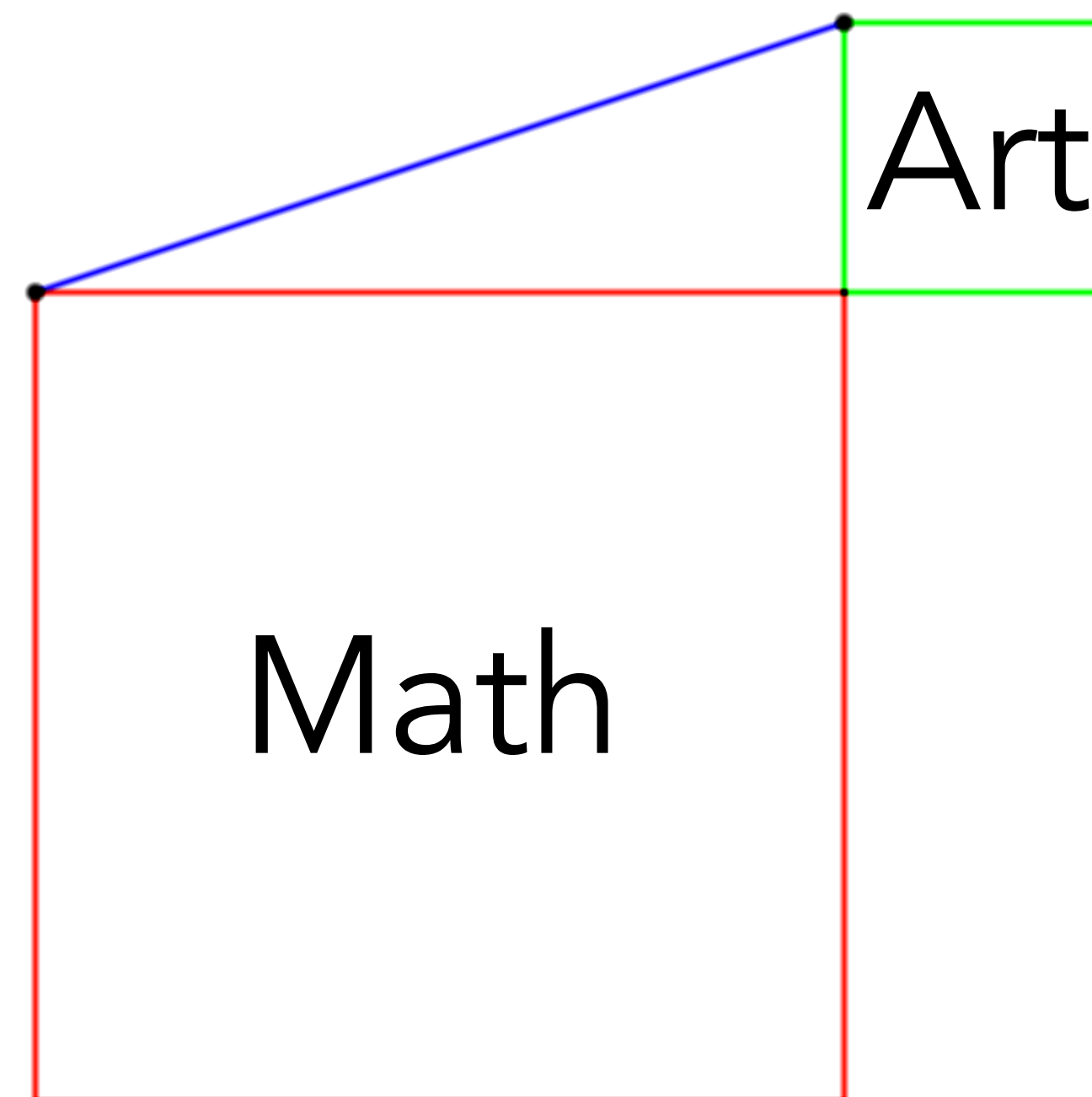


That blue line isn't one or the other.

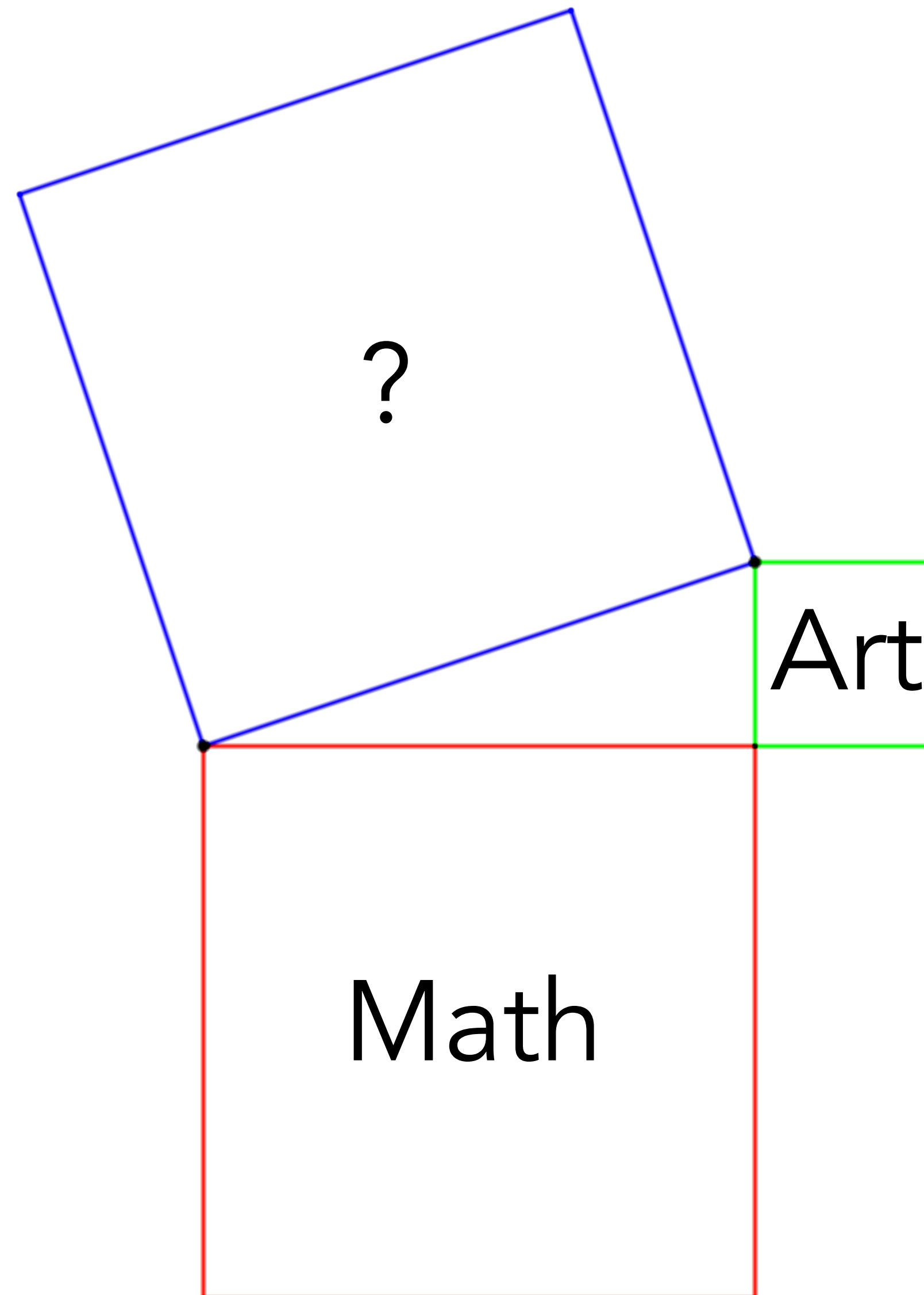






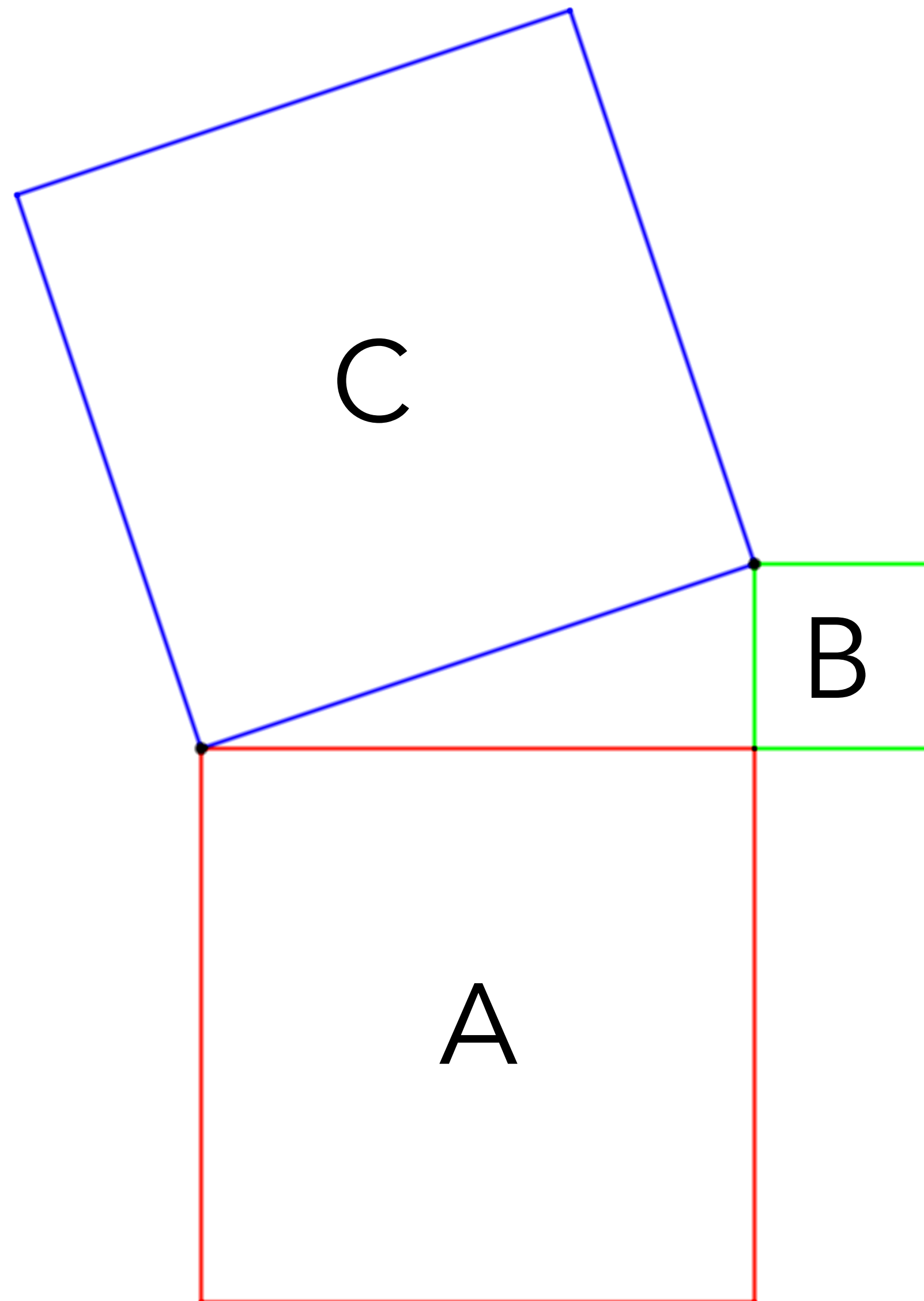






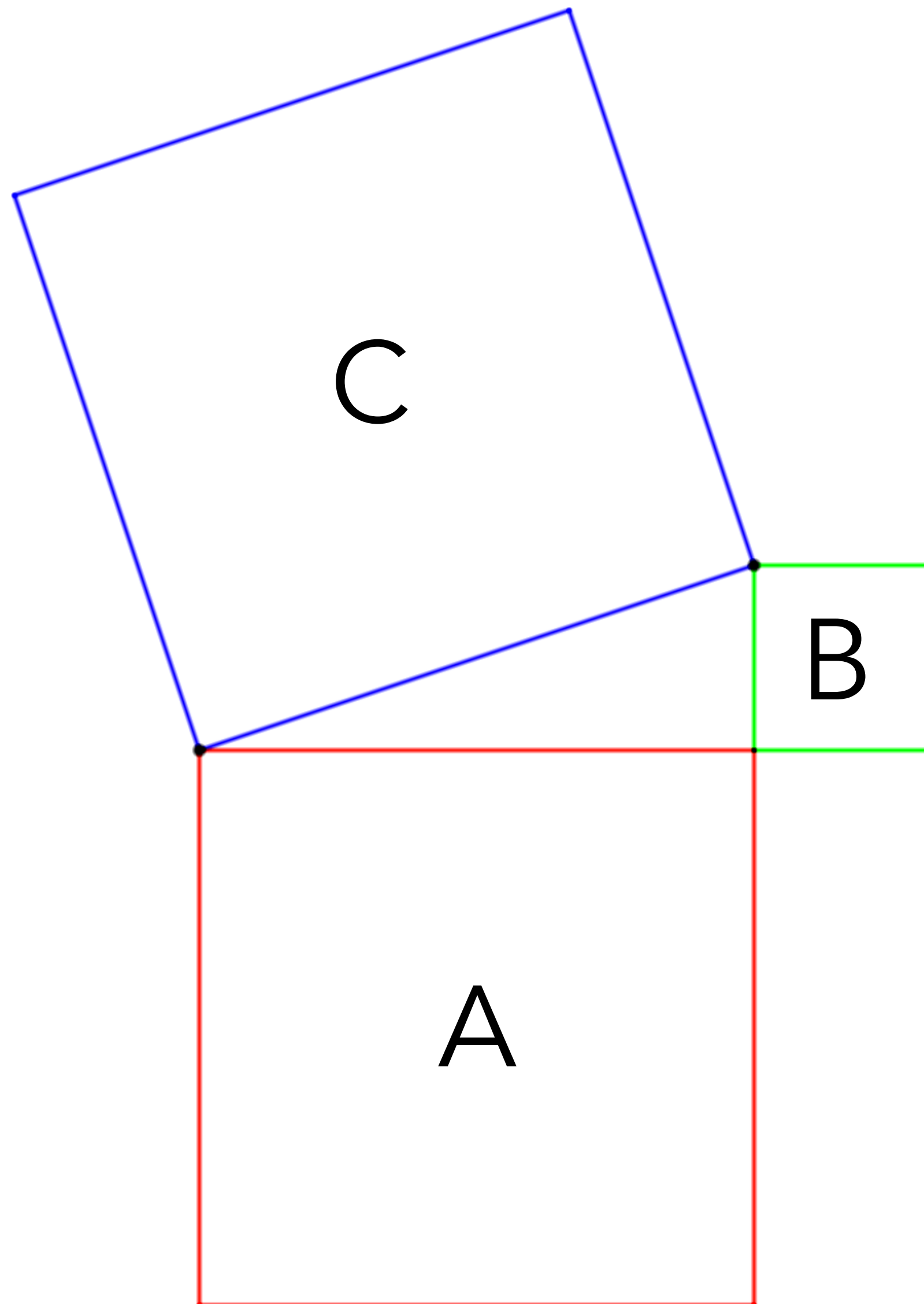


$$A^*A + B^*B = C^*C$$



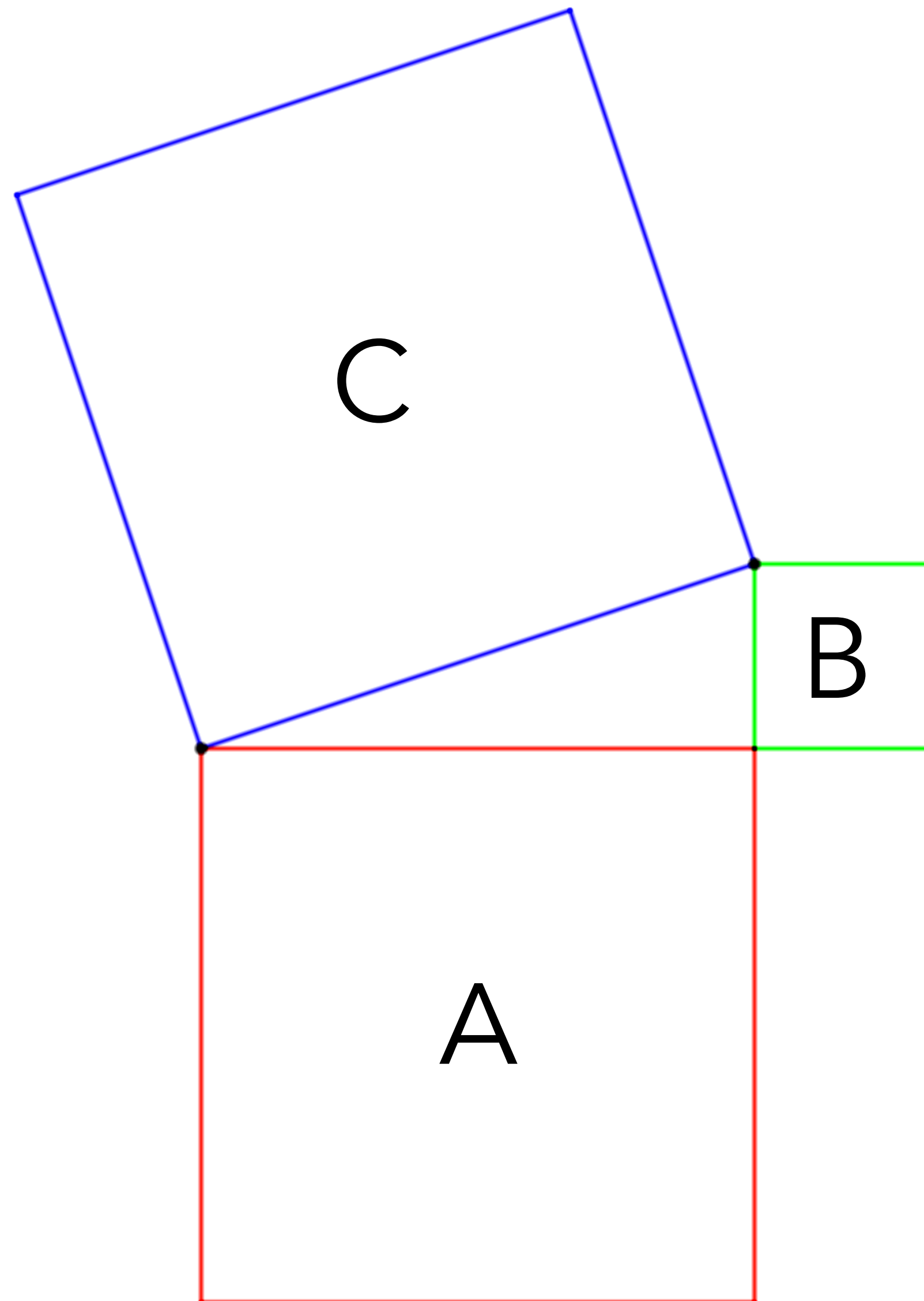


$$\text{sqrt}(A * A + B * B) = C$$

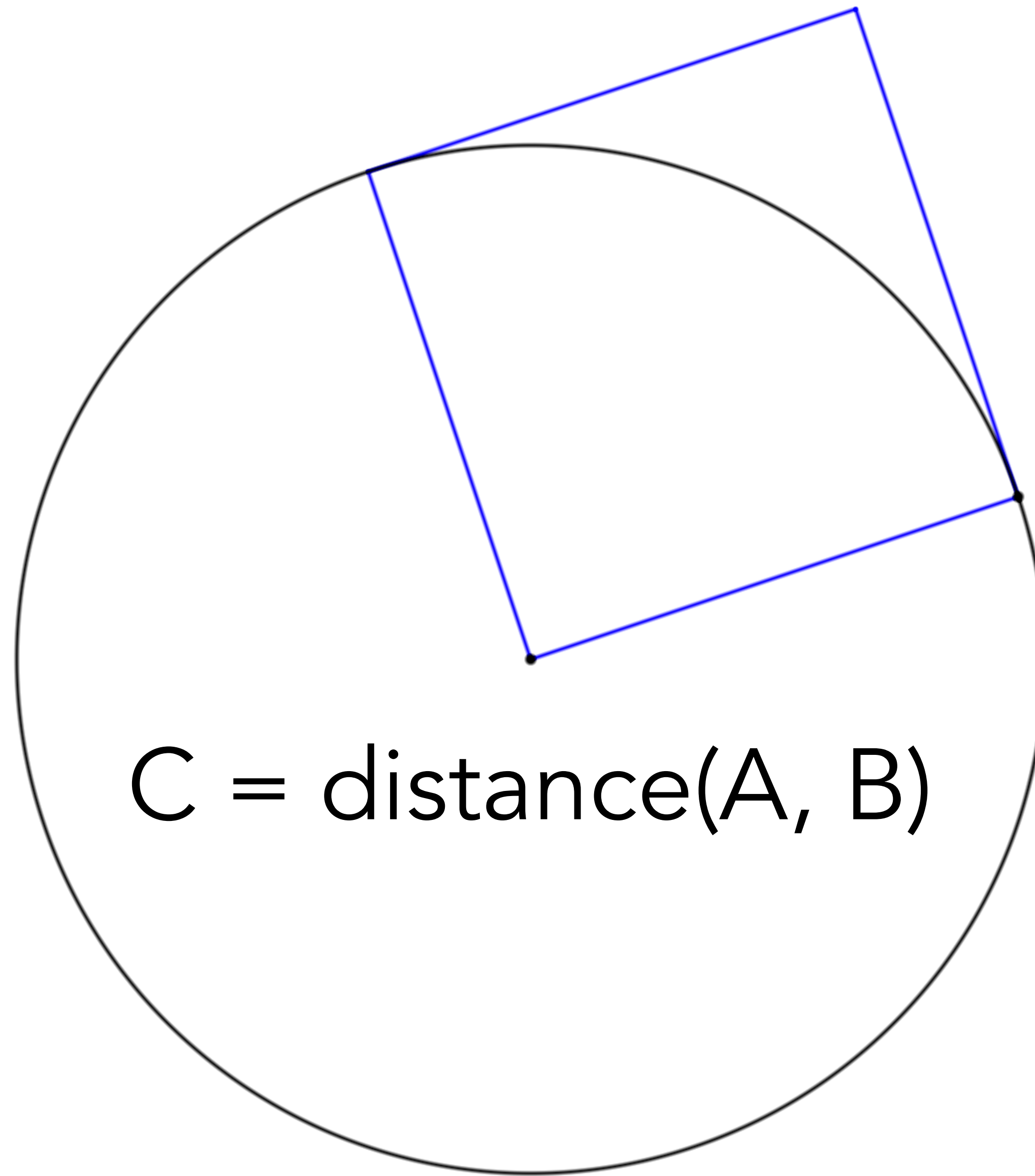




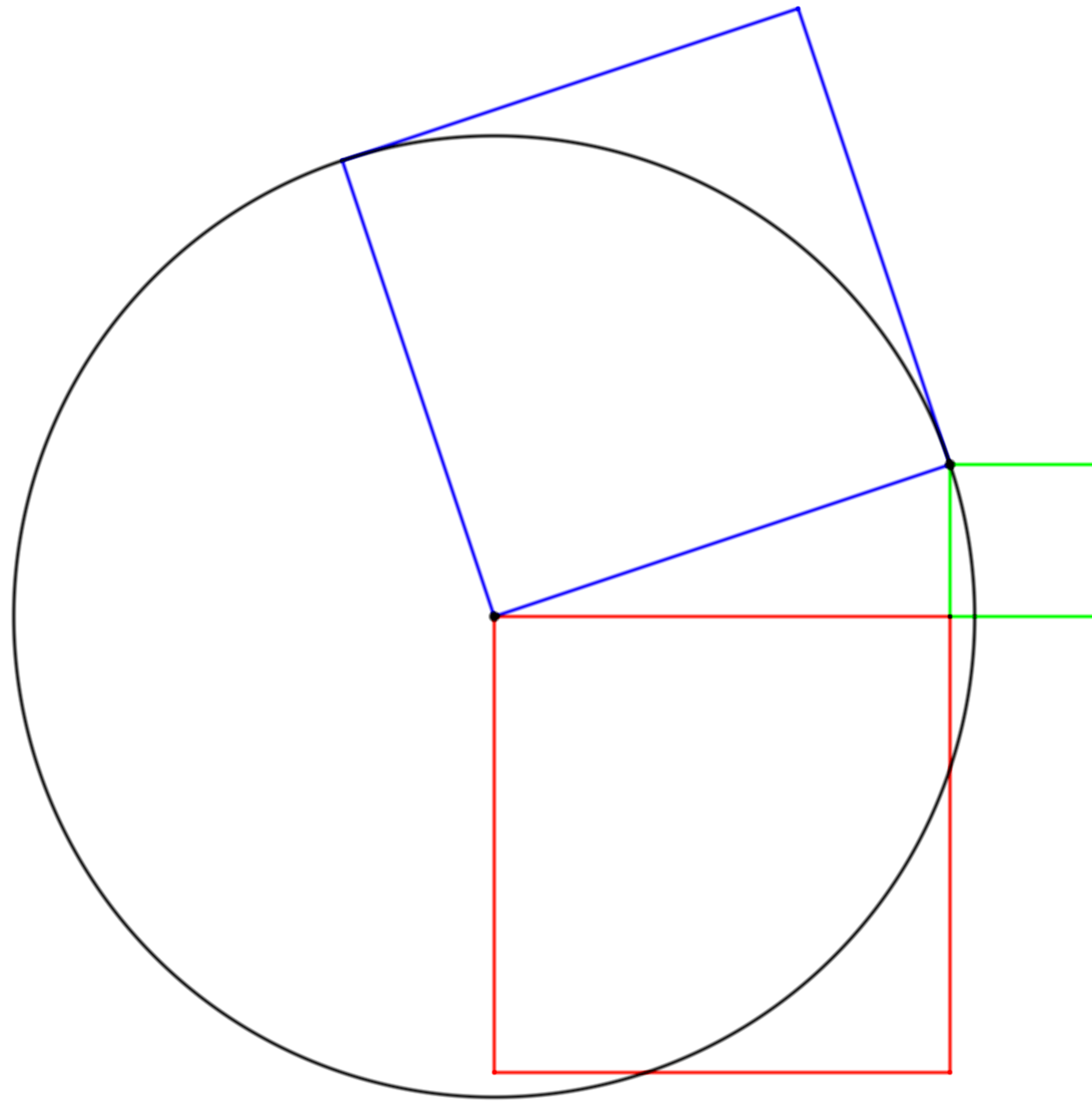
$$C = \text{distance}(A, B)$$





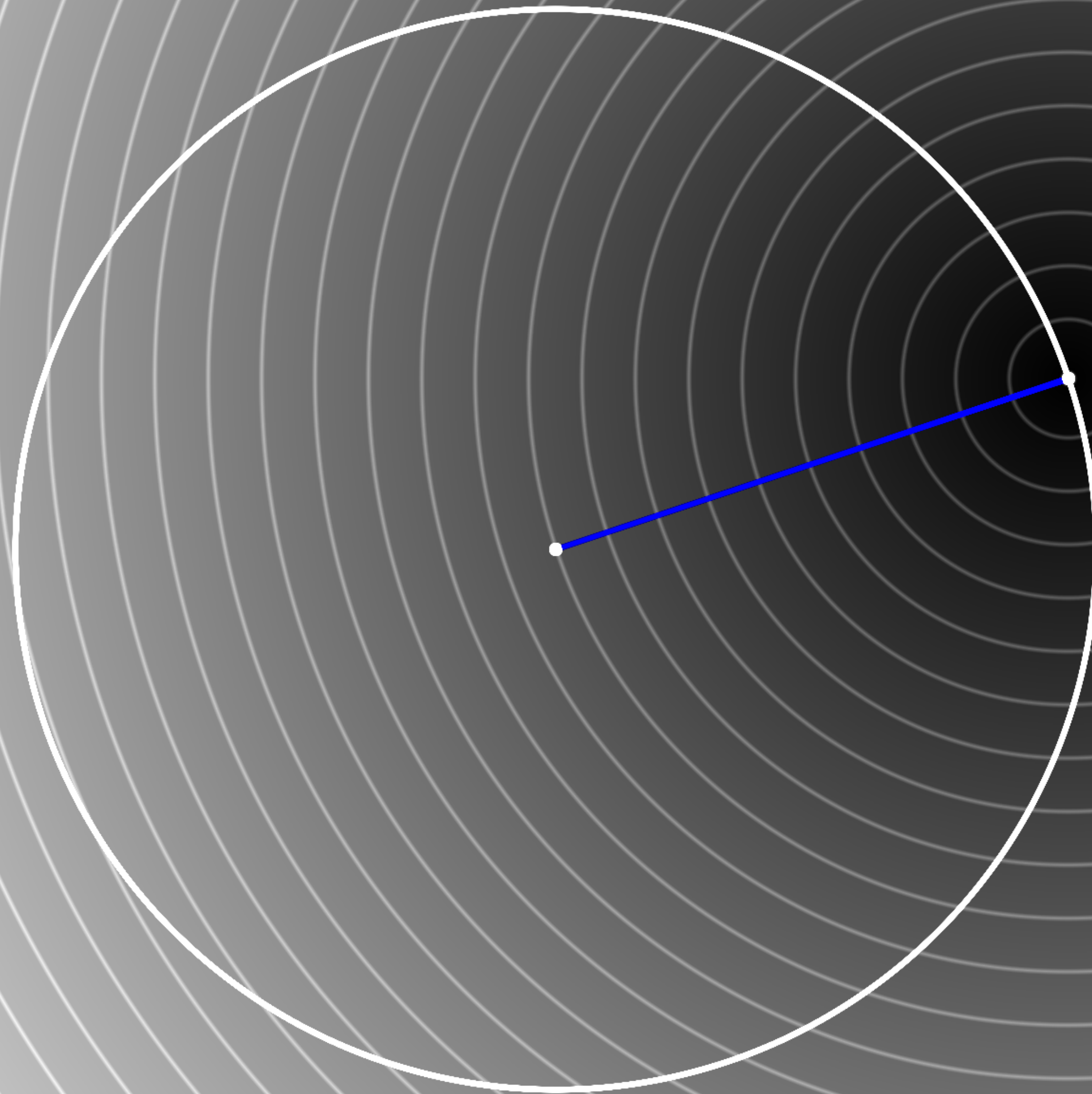






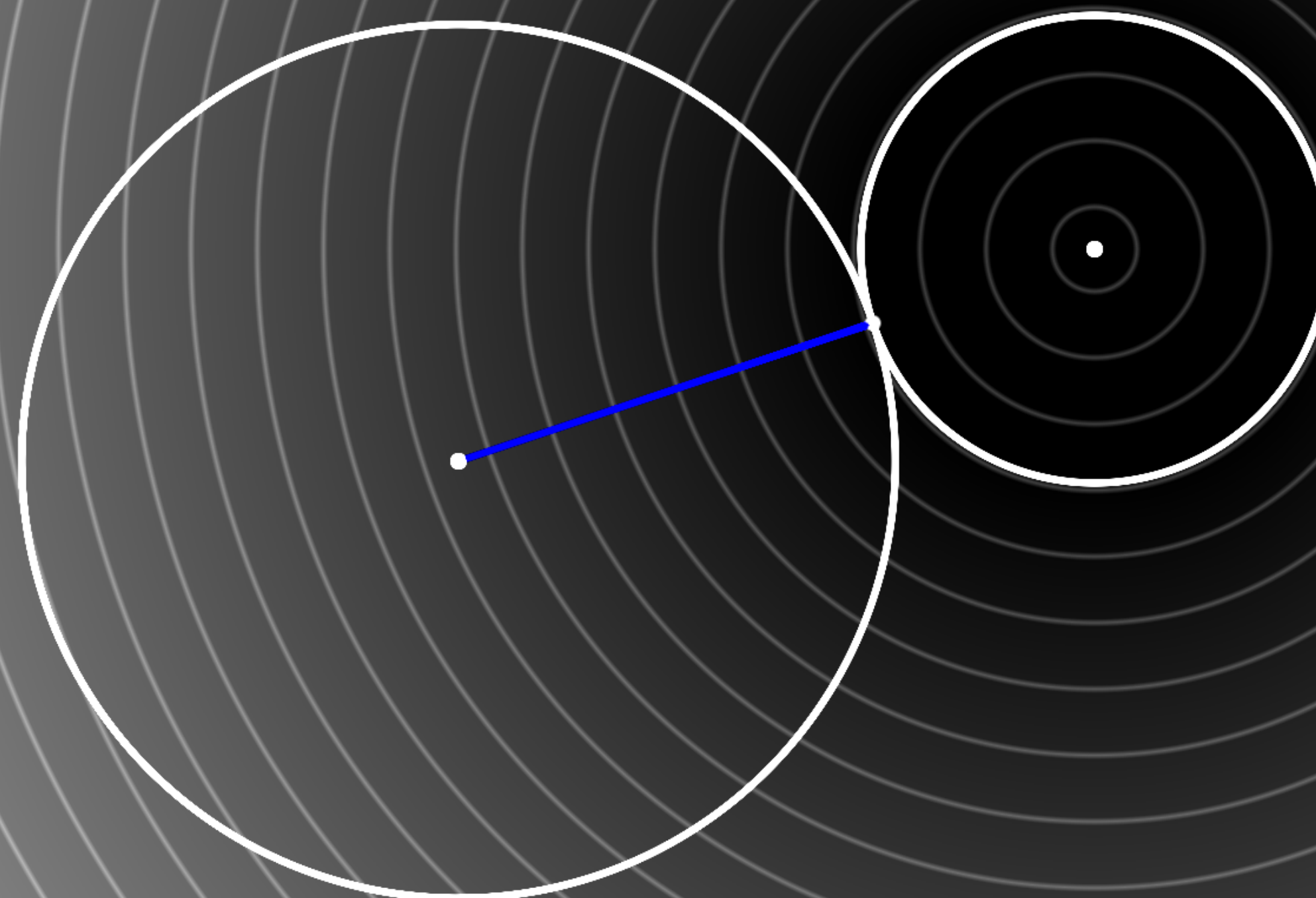
This circle is the same distance in every direction.





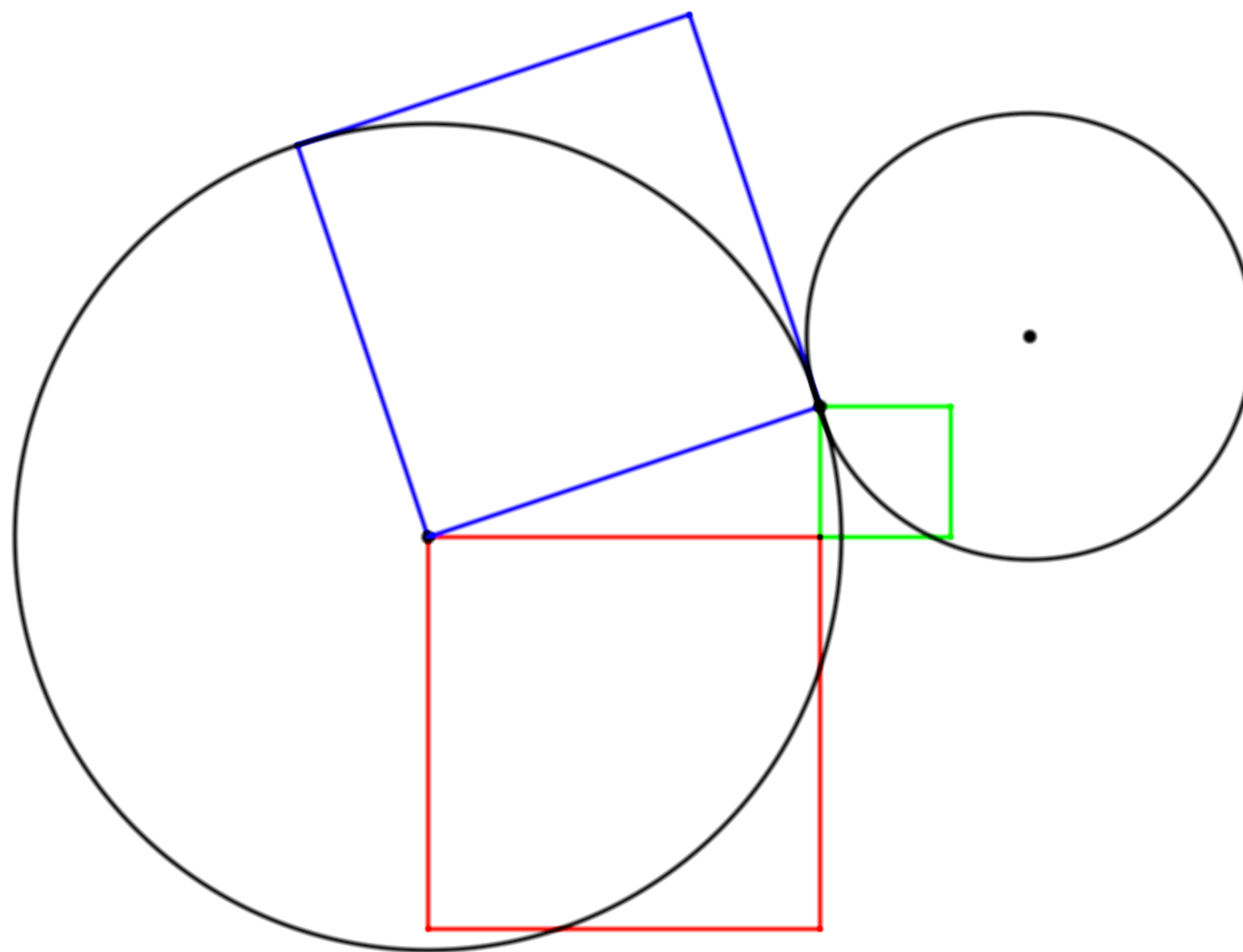
The distance field.



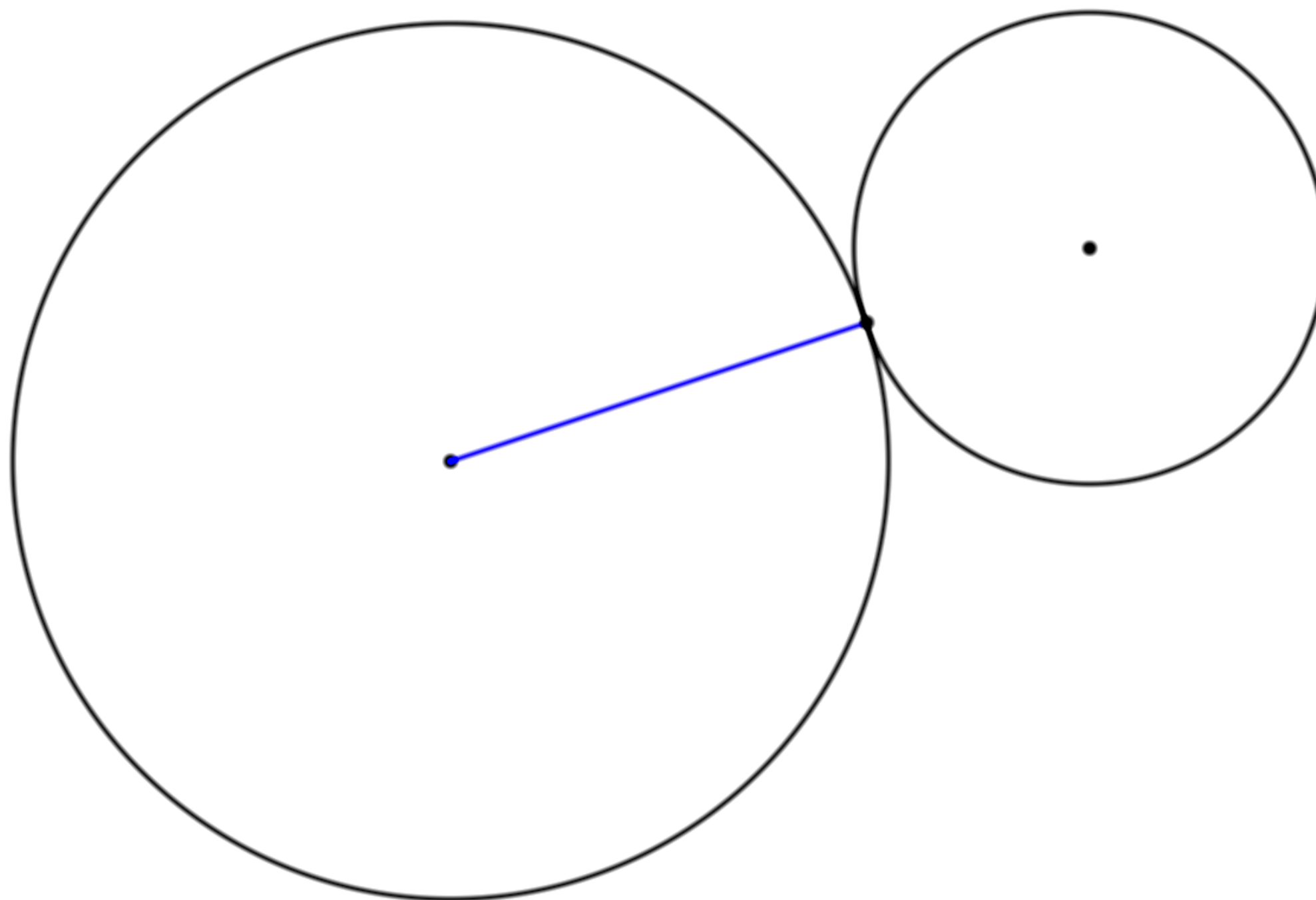


Subtract a radius, and the distance field is now a circle.

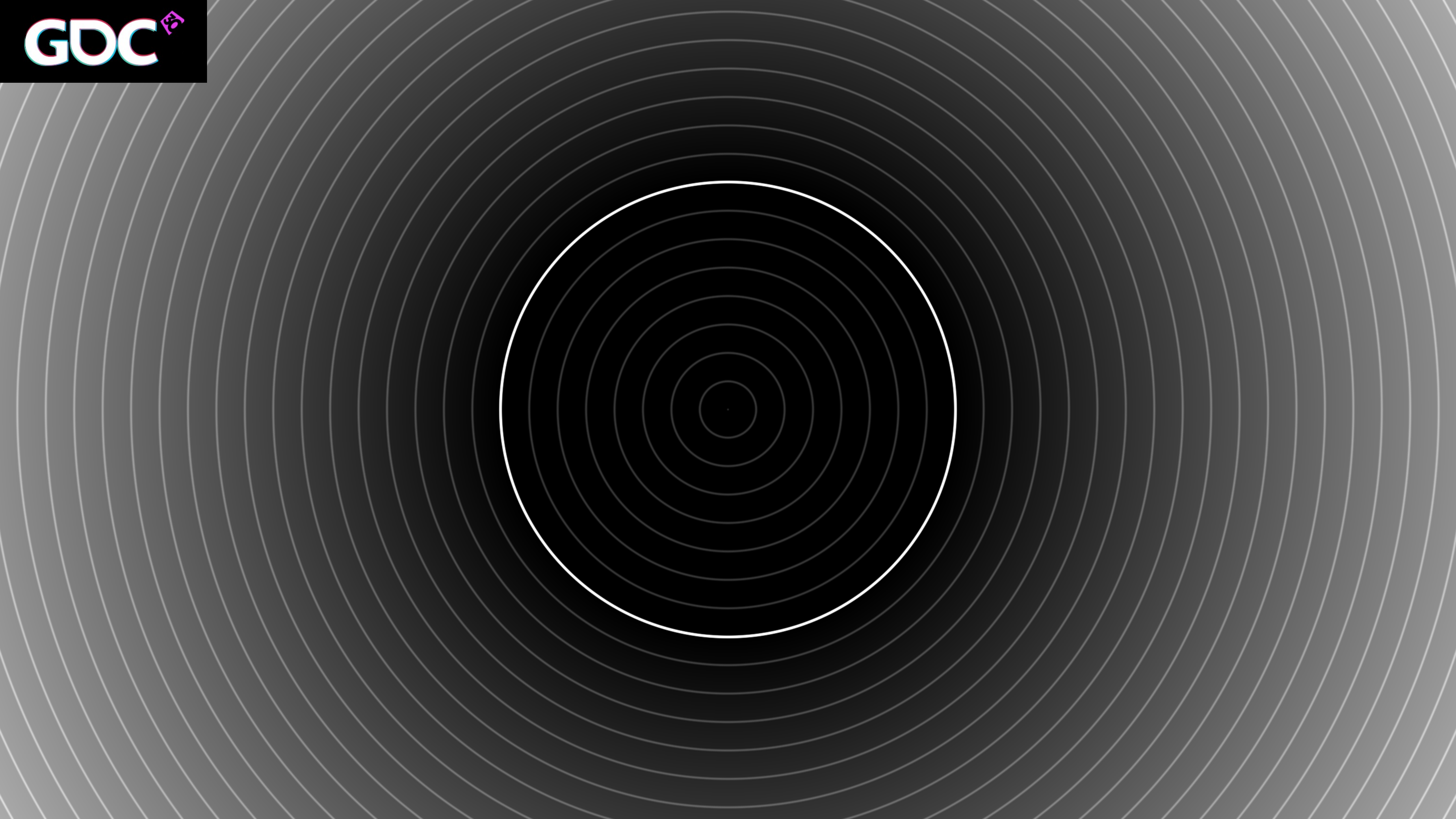














```
float sphere(vec2 position, float radius)
{
    return length(position)-radius;
}
```









```
float y(float height)
{
    return position.y - height;
}
```

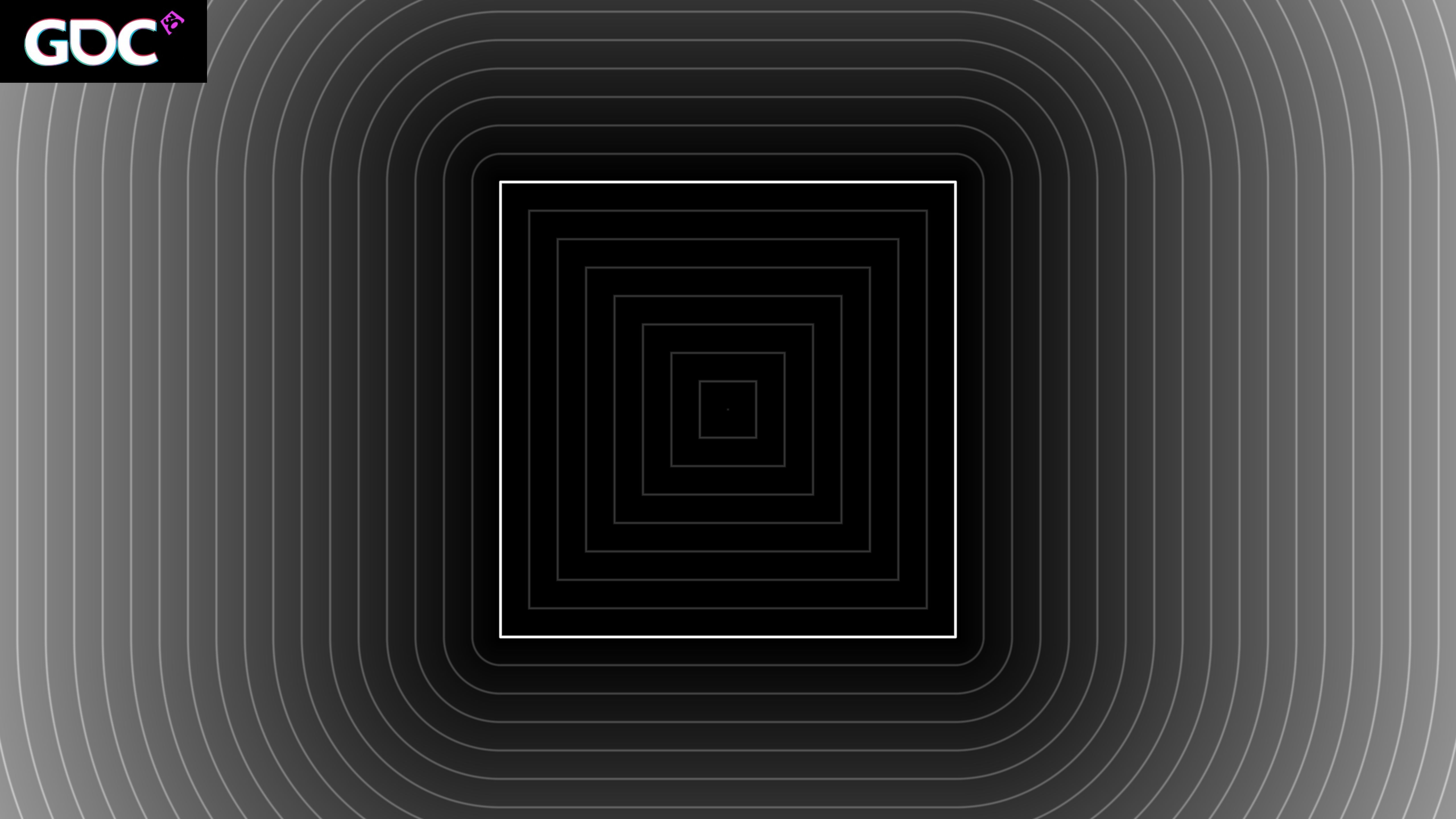






```
float x(float offset)
{
    return position.y - offset;
}
```



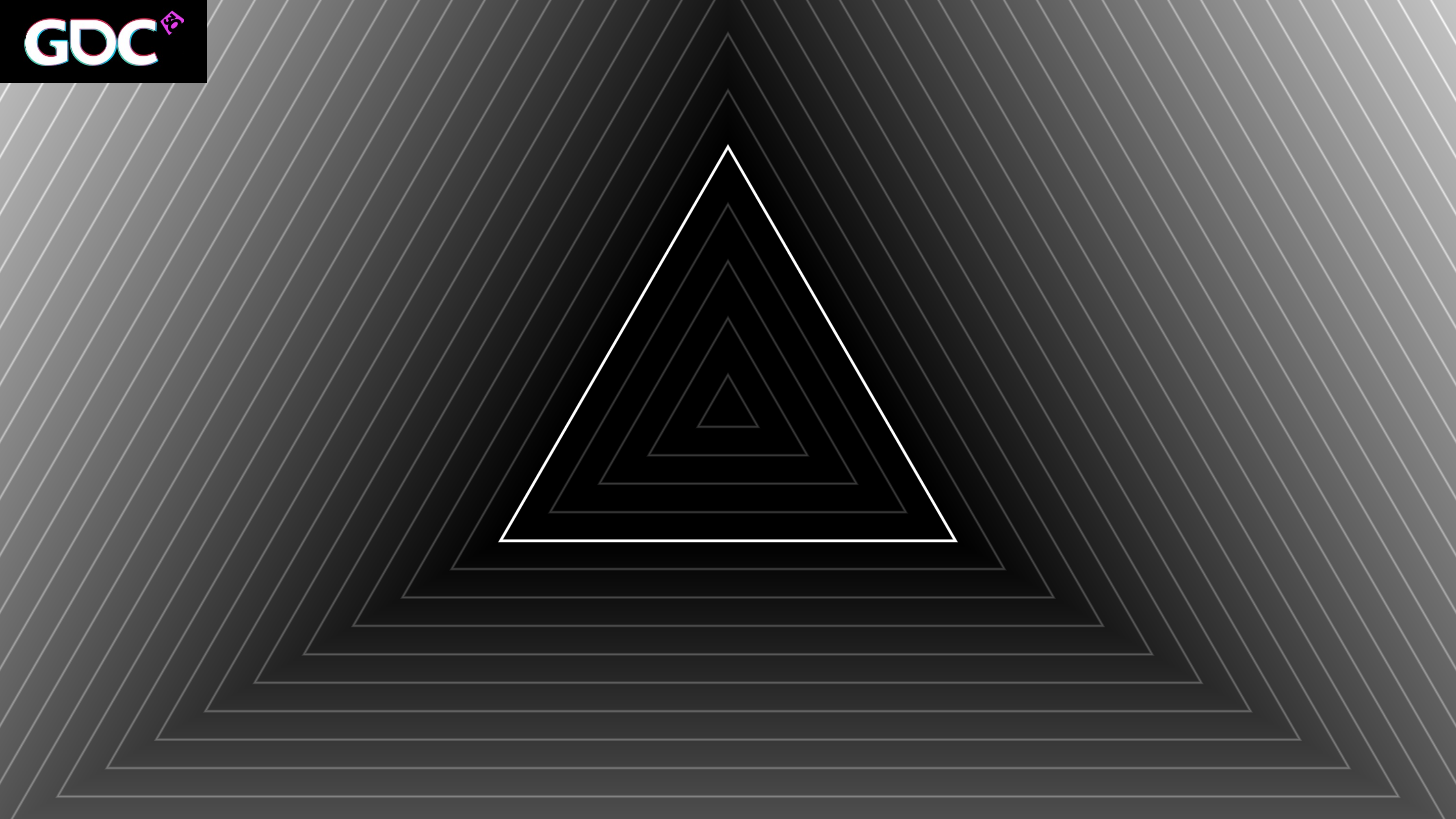




```
float cube(vec2 position, vec2 scale)
{
    vec2 vertex    = abs(position) - scale;
    vec2 edge      = max(vertex, 0.);
    float interior  = max(vertex.x, vertex.y);

    return min(interior, 0.) + length(edge);
}
```





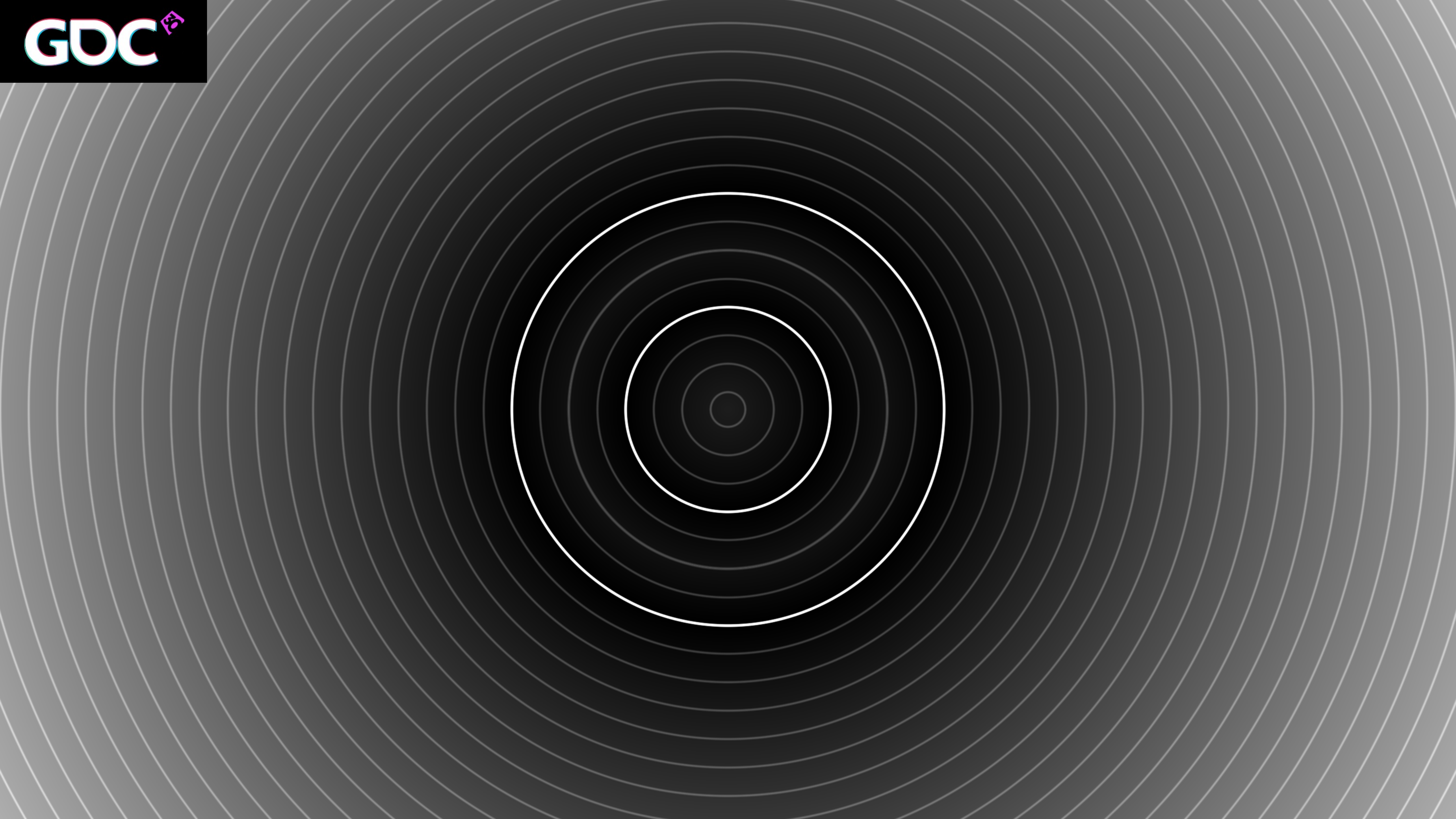


```
float prism(vec2 position, float scale)
{
    position.y    *= 0.57735026918; //1./sqrt(3.);
    scale         *= 0.57735026918;

    vec3 edge     = vec3(0.);
    edge.x        = position.y + position.x;
    edge.y        = position.x - position.y;
    edge.z        = position.y + position.y;
    edge          *= .86602540358; //sqrt(3.)/2.;

    return max(edge.x, max(-edge.y, -edge.z))-scale;
}
```

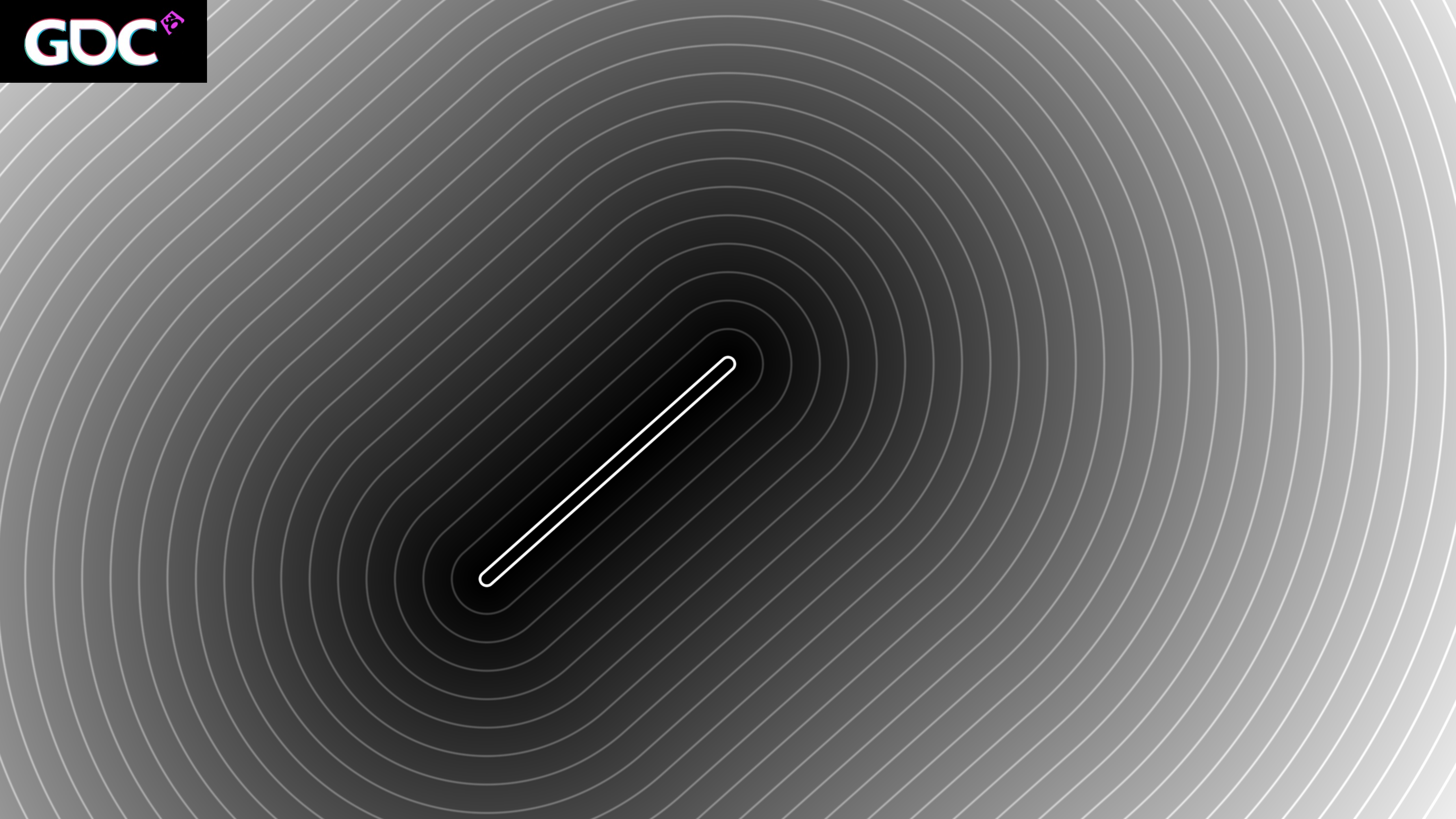






```
float torus(vec2 position, vec2 radius)
{
    return abs(abs(length(position)-radius.x)-radius.y);
}
```

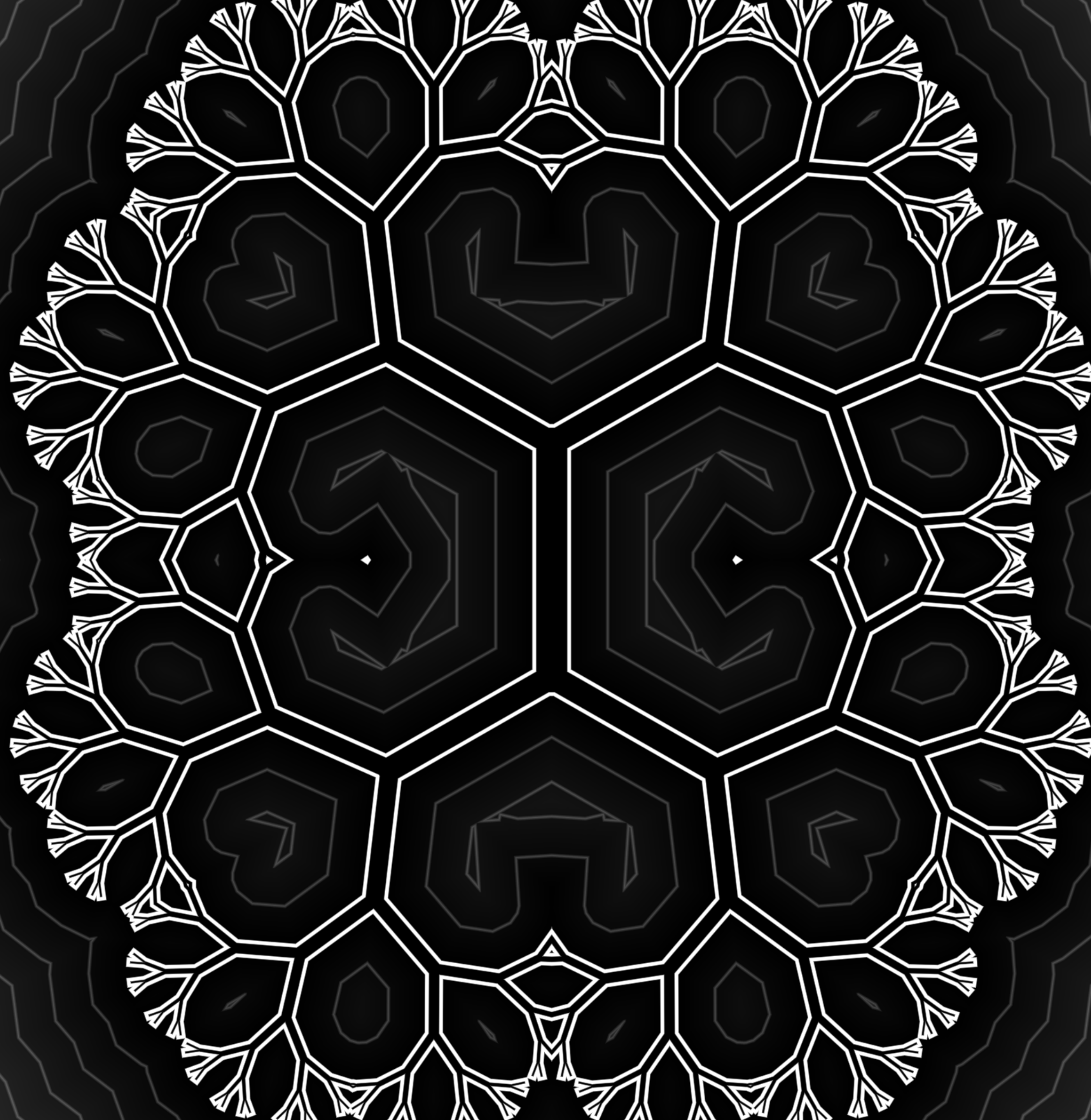






```
float torus(vec2 position, vec2 radius)
{
    return abs(abs(length(position)-radius.x)-radius.y);
}
```

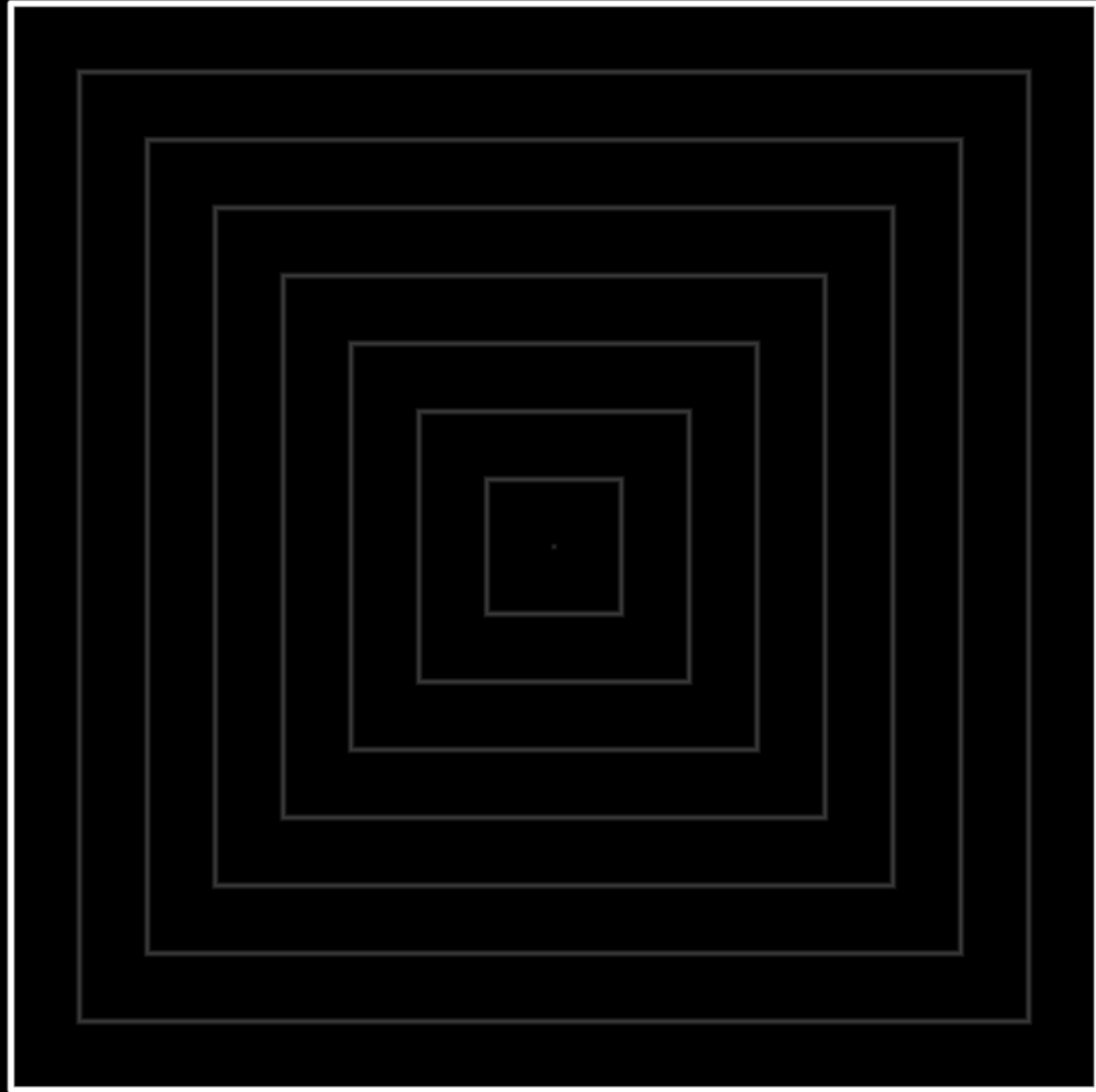






```
float fractal(vec2 position, float rotation, float translation, float scale)
{
    const int iterations      = 8;
    float radius              = 1.5/float(iterations);
    rotation                  = rotation * TAU + TAU * .5;
    position                   = abs(position);
    position.y                -= radius;
    scale                      *= radius;
    float result              = max(position.x, position.y) - scale;
    for (int i = 0; i < iterations; i++)
    {
        float magnitude       = length(position);
        float phase            = atan(position.x, position.y);
        phase                  = mod(phase, rotation) - rotation * .5;
        position.x             = magnitude * cos(phase);
        position.y             = magnitude * sin(phase);
        position               = abs(position);
        position.y             -= translation;
        translation            *= .7;
        scale                  *= .8;
        rotation                *= -.98;
        result                  = min(max(position.x, position.y) - scale, result);
    }
    return result;
}
```

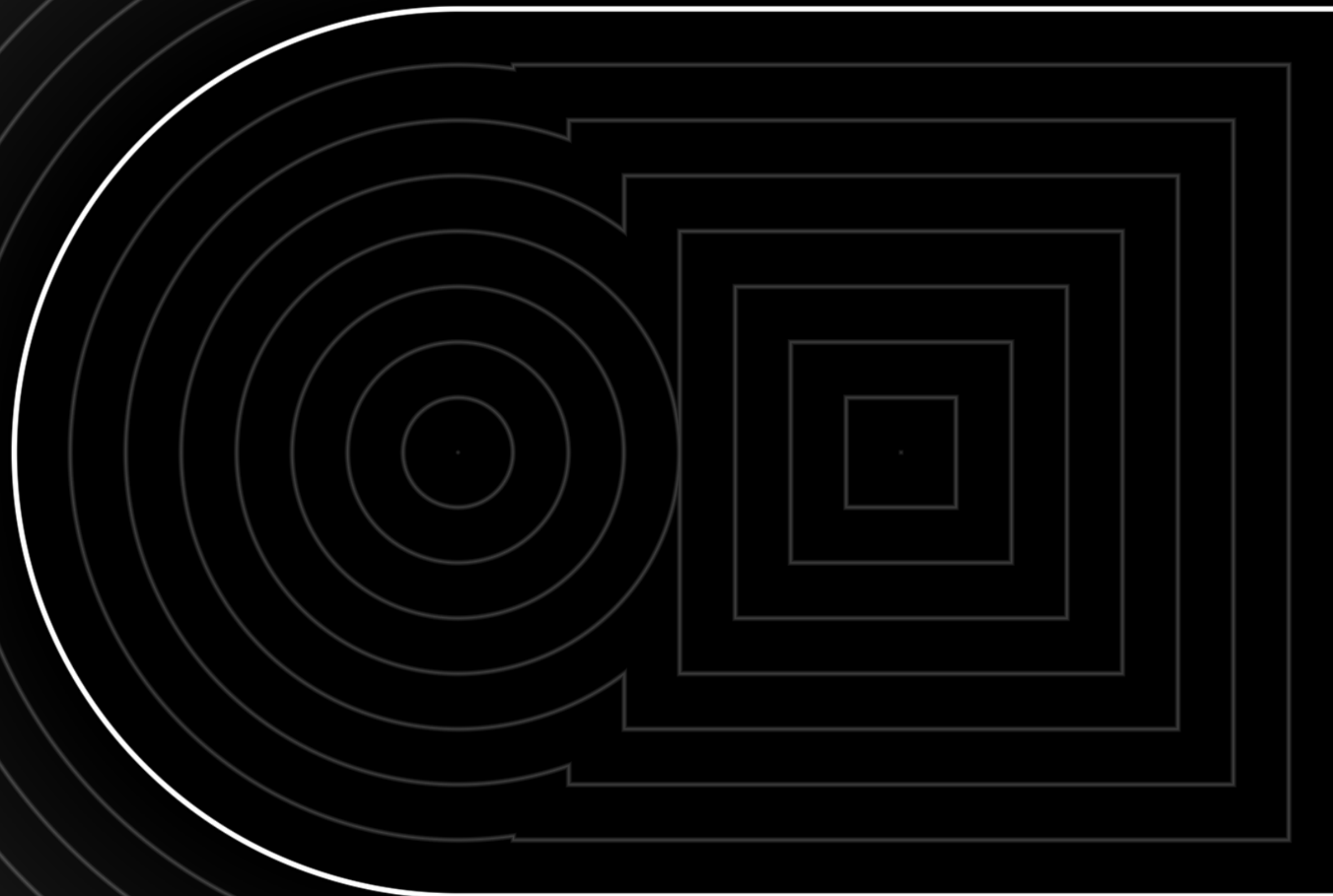






`min(a, b)`

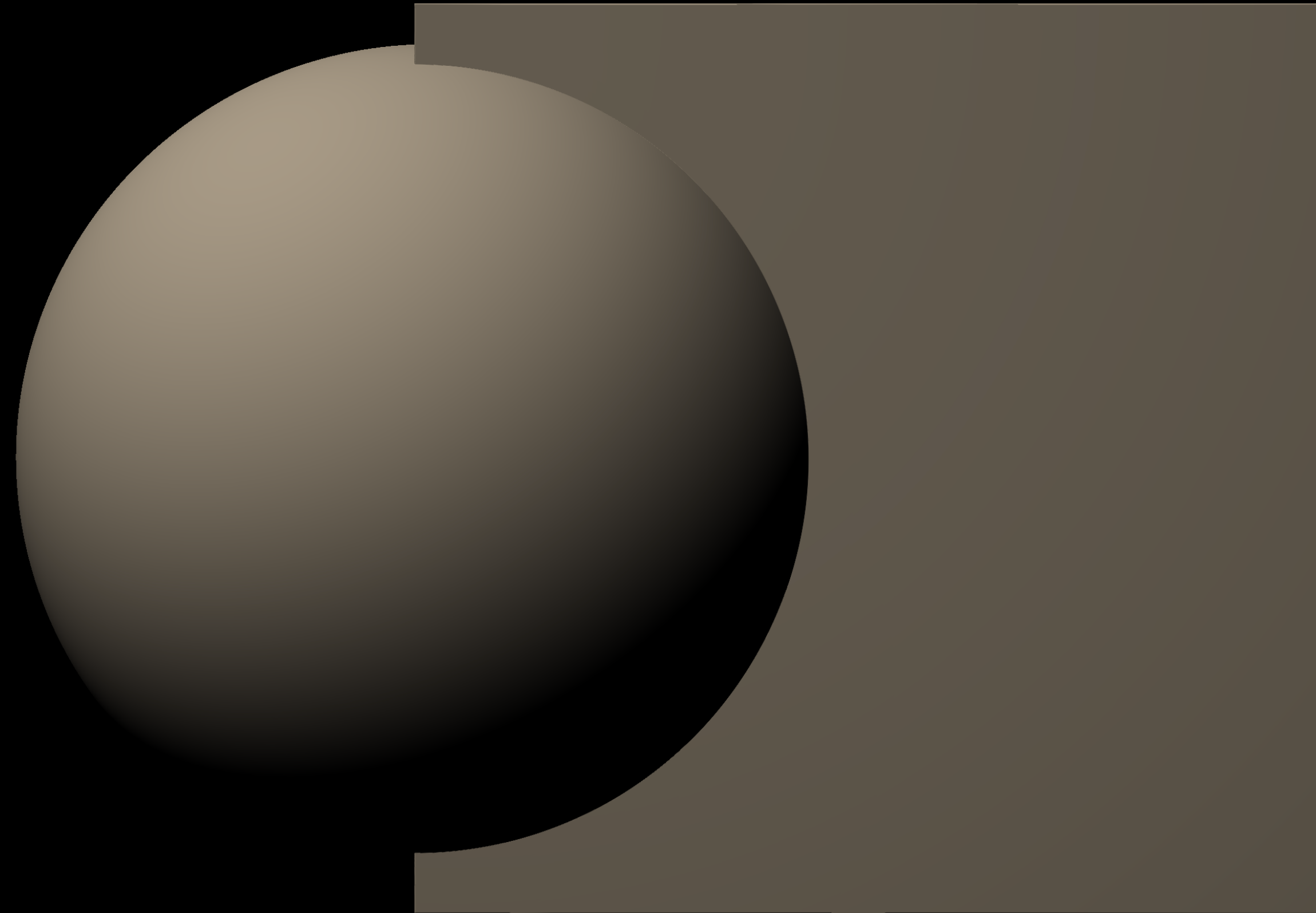






`min(a, b)`









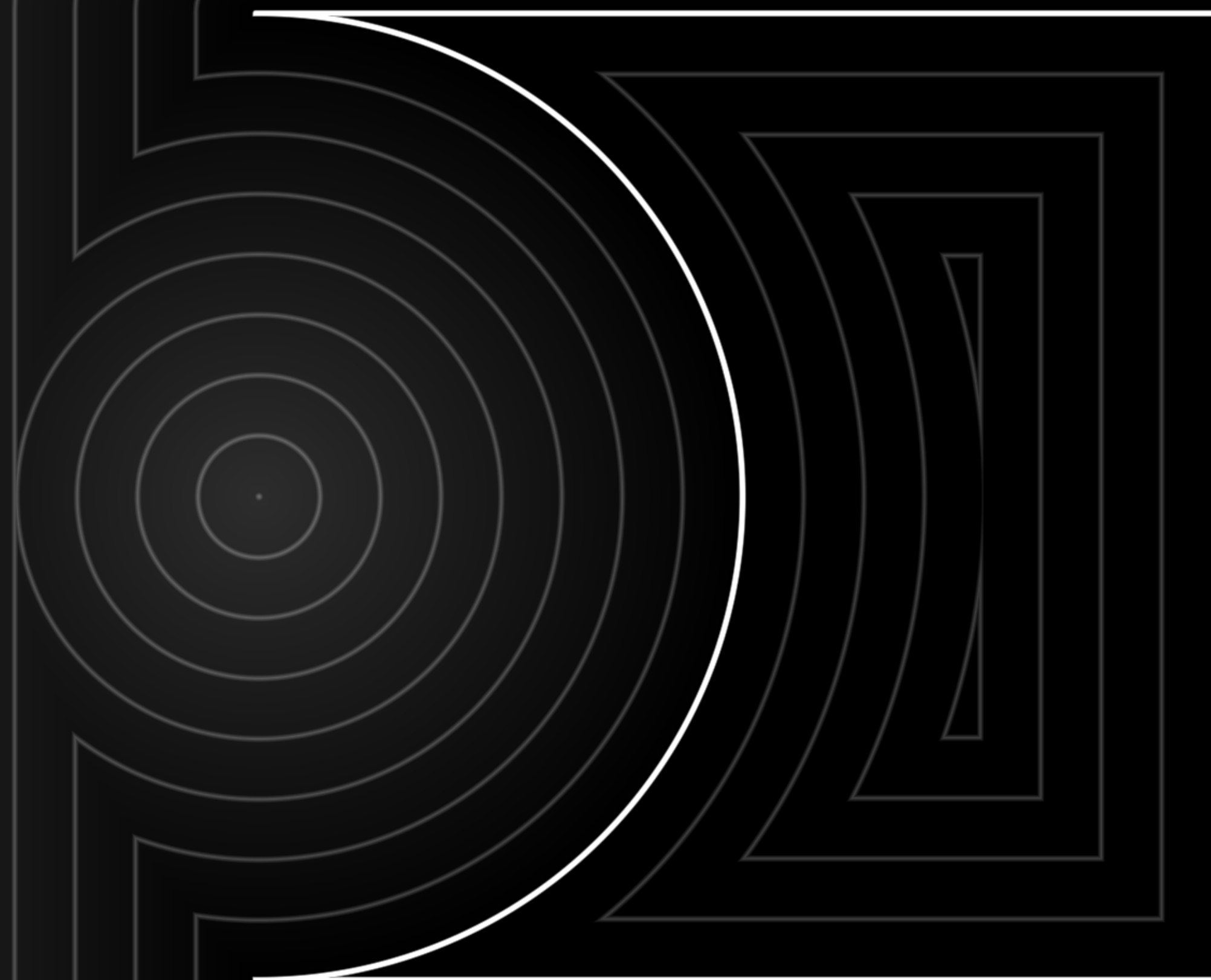


`max(a, b)`





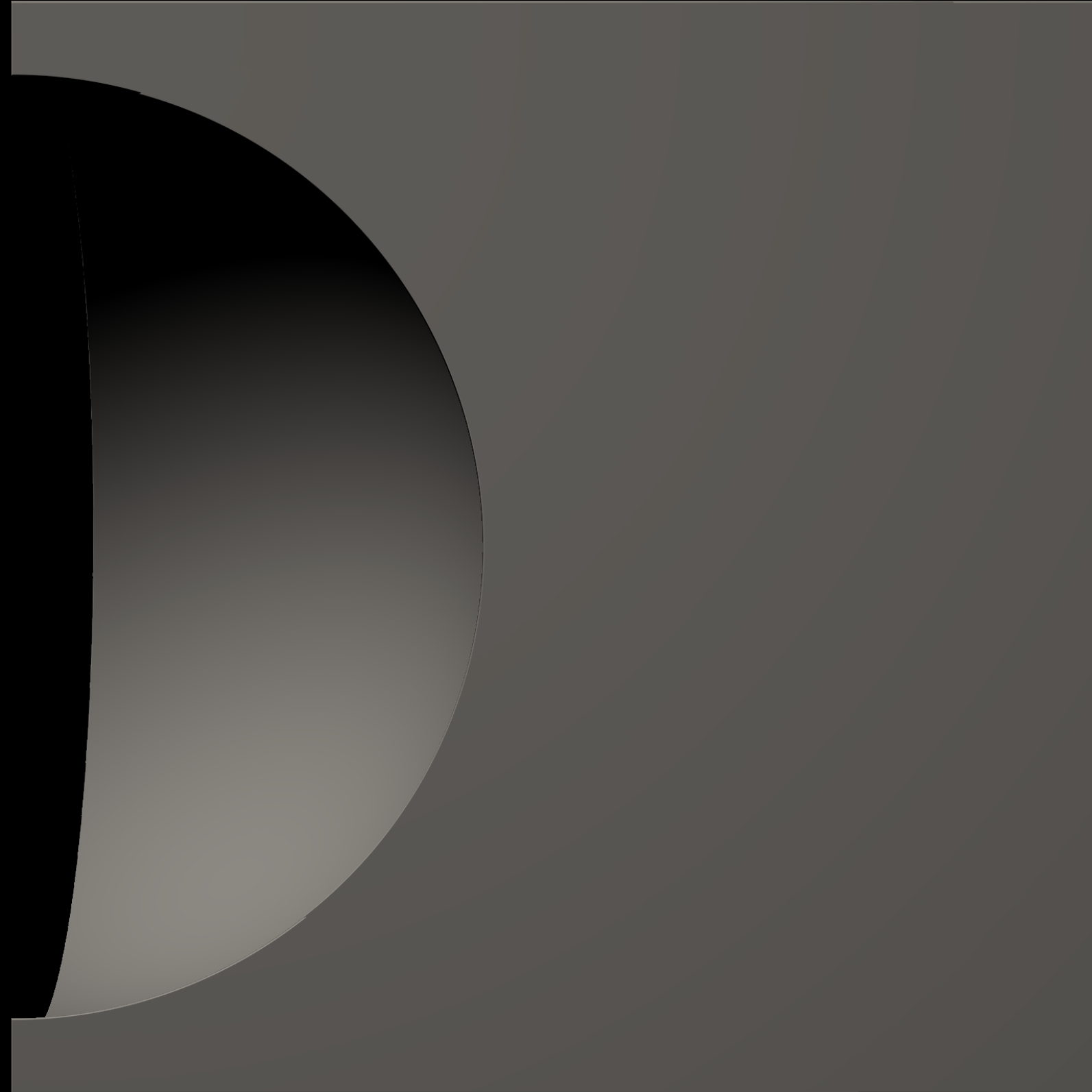




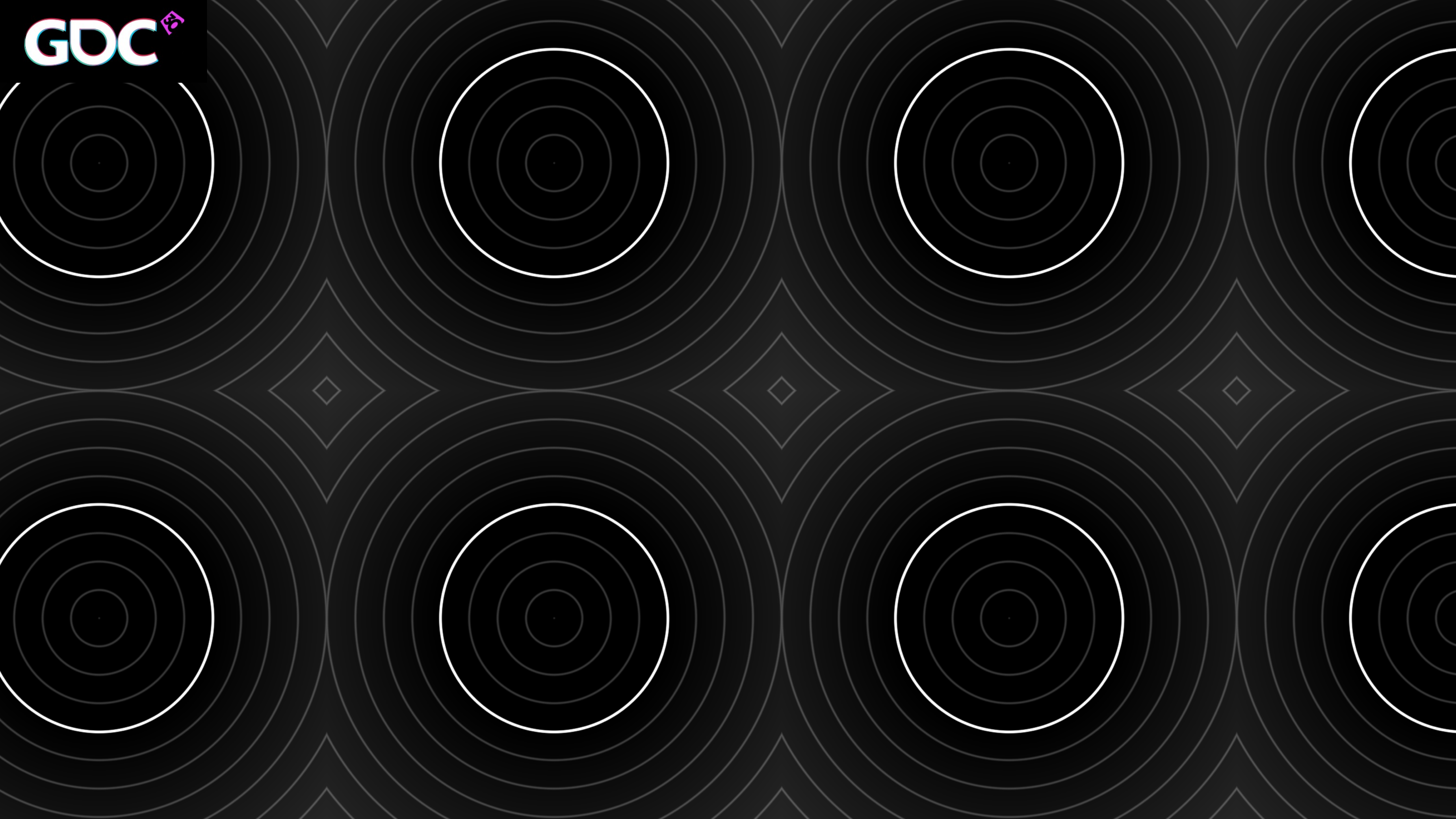


$\max(-a, b)$





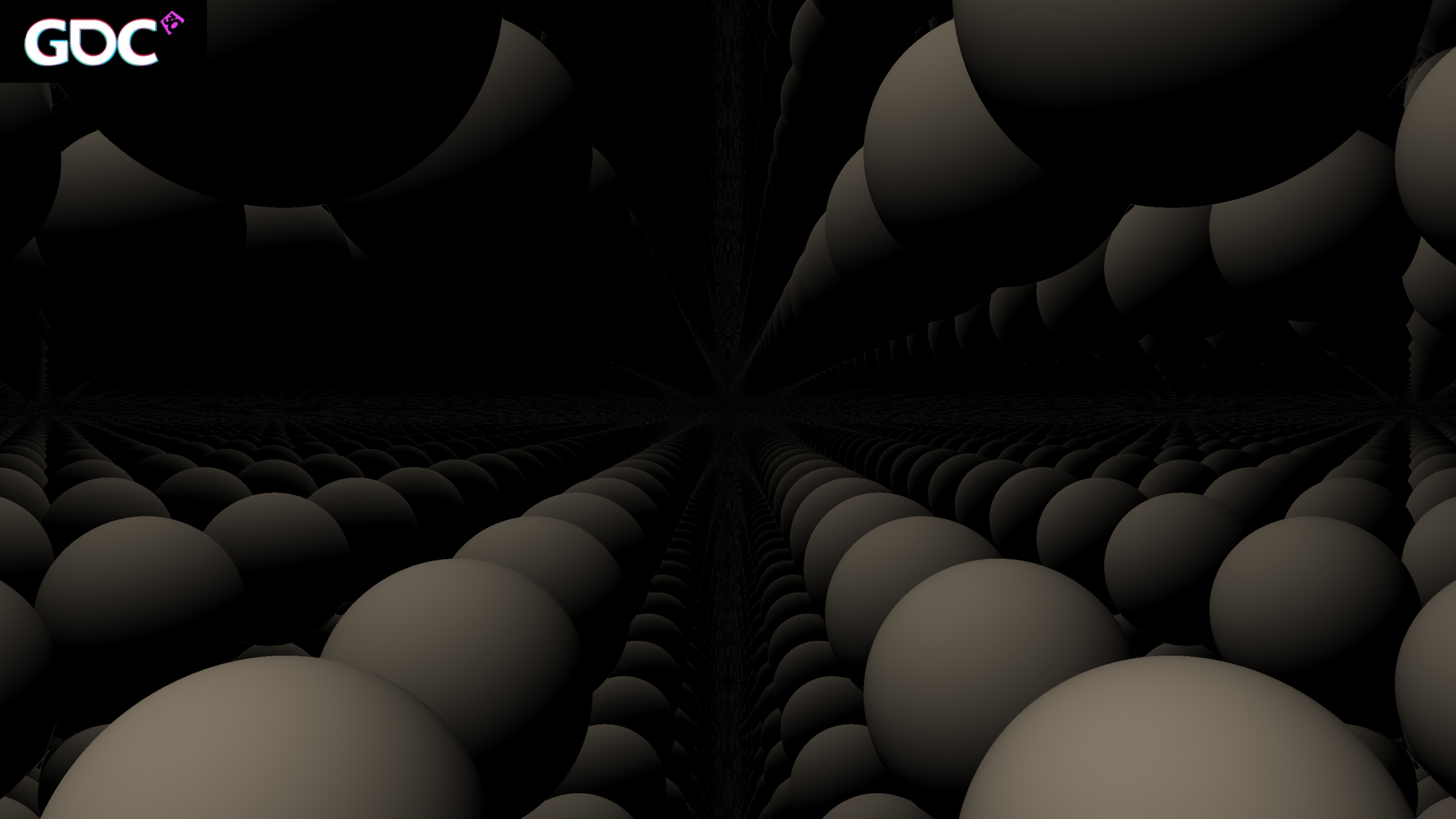




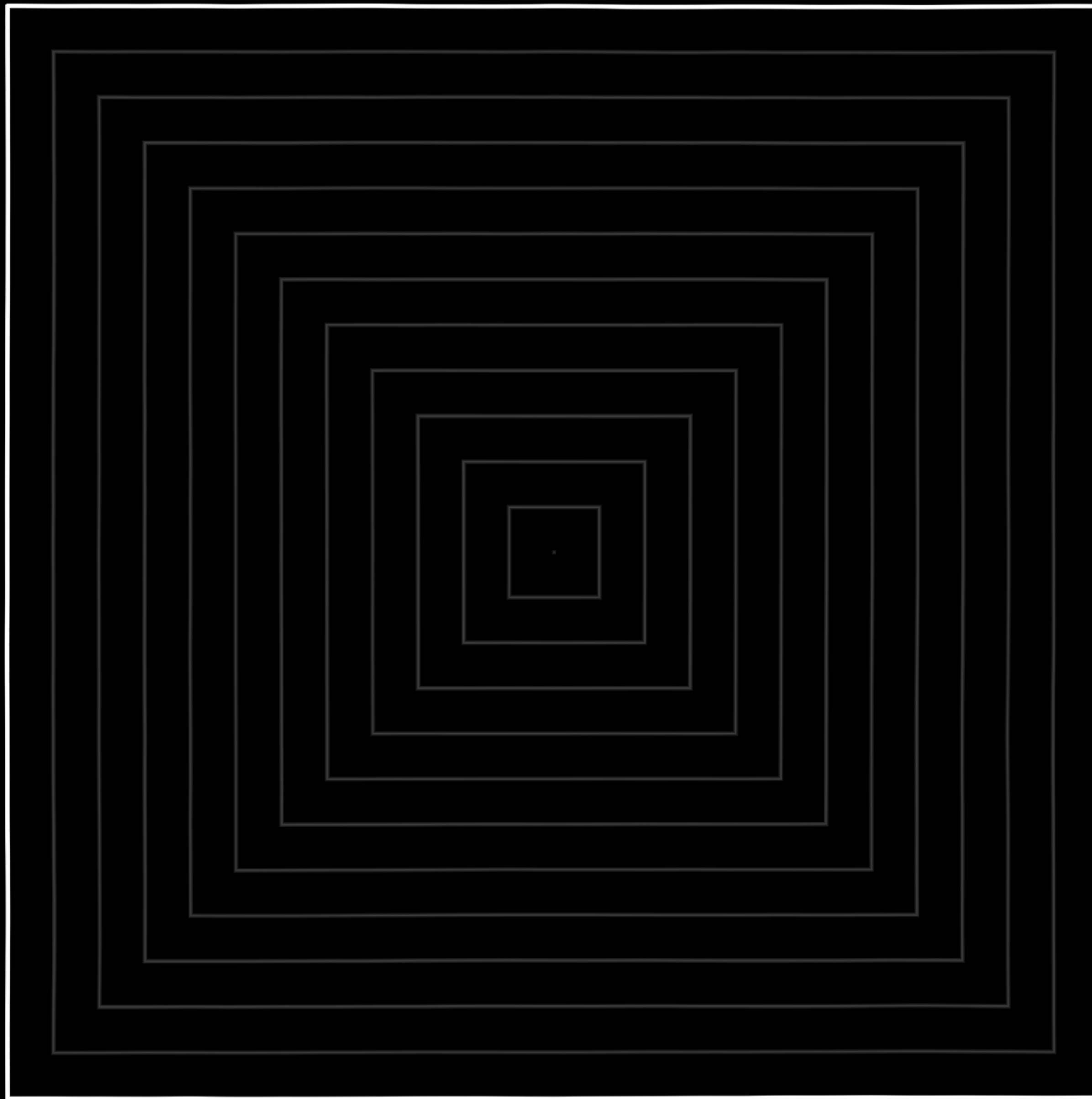


```
mod(position, scale) - scale * .5
```

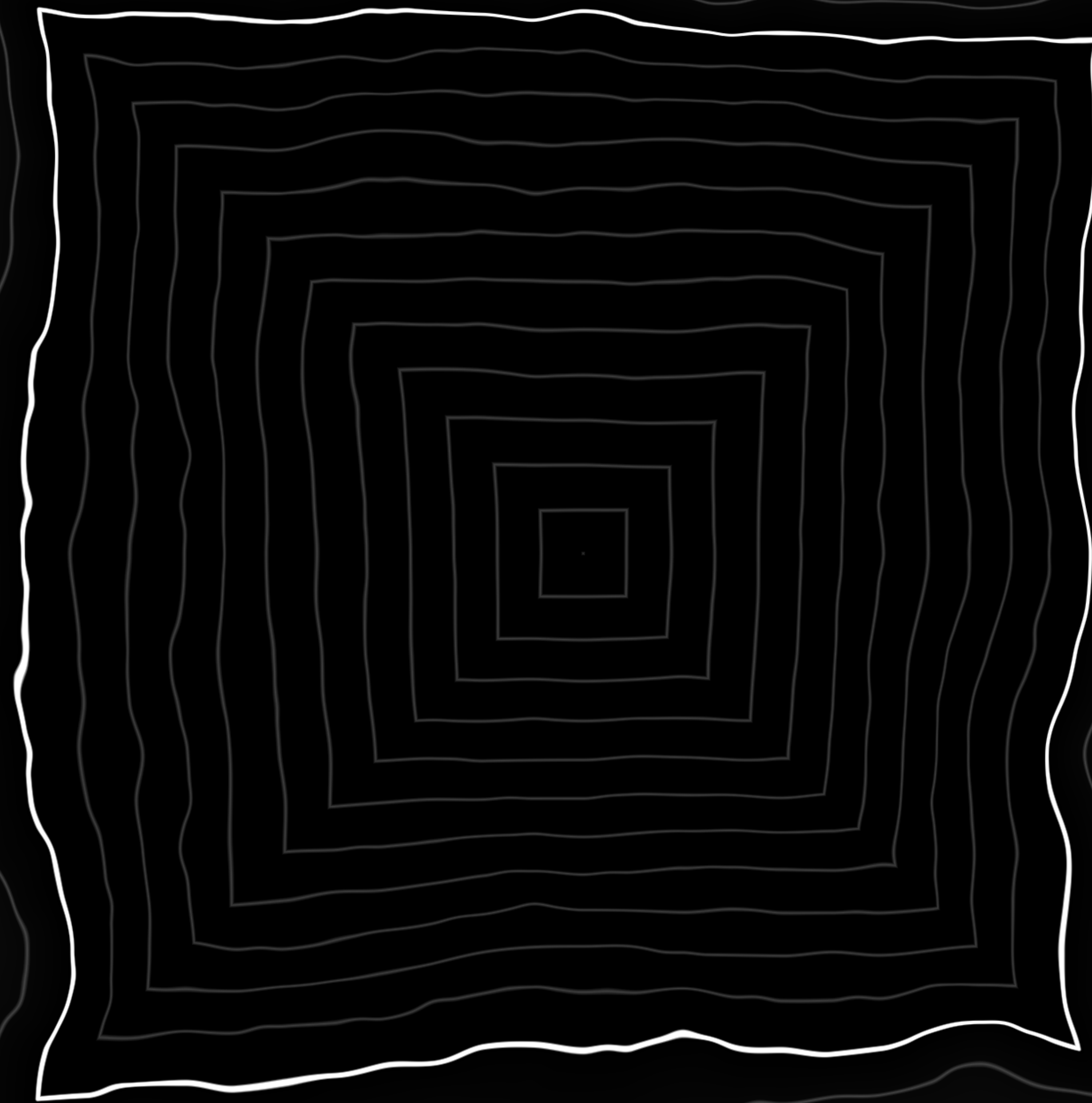














```
float noise(in vec2 uv)
{
    const float k          = 257.;
    vec4 l                 = vec4(floor(uv),fract(uv));
    l.zw                   = l.zw*l.zw*(3.-2.*l.zw);

    float u                = l.x + l.y * k;

    vec4 v                 = vec4(u, u+1.,u+k, u+k+1.);
    v                      = fract(fract(1.23456789*v)*v/.987654321);

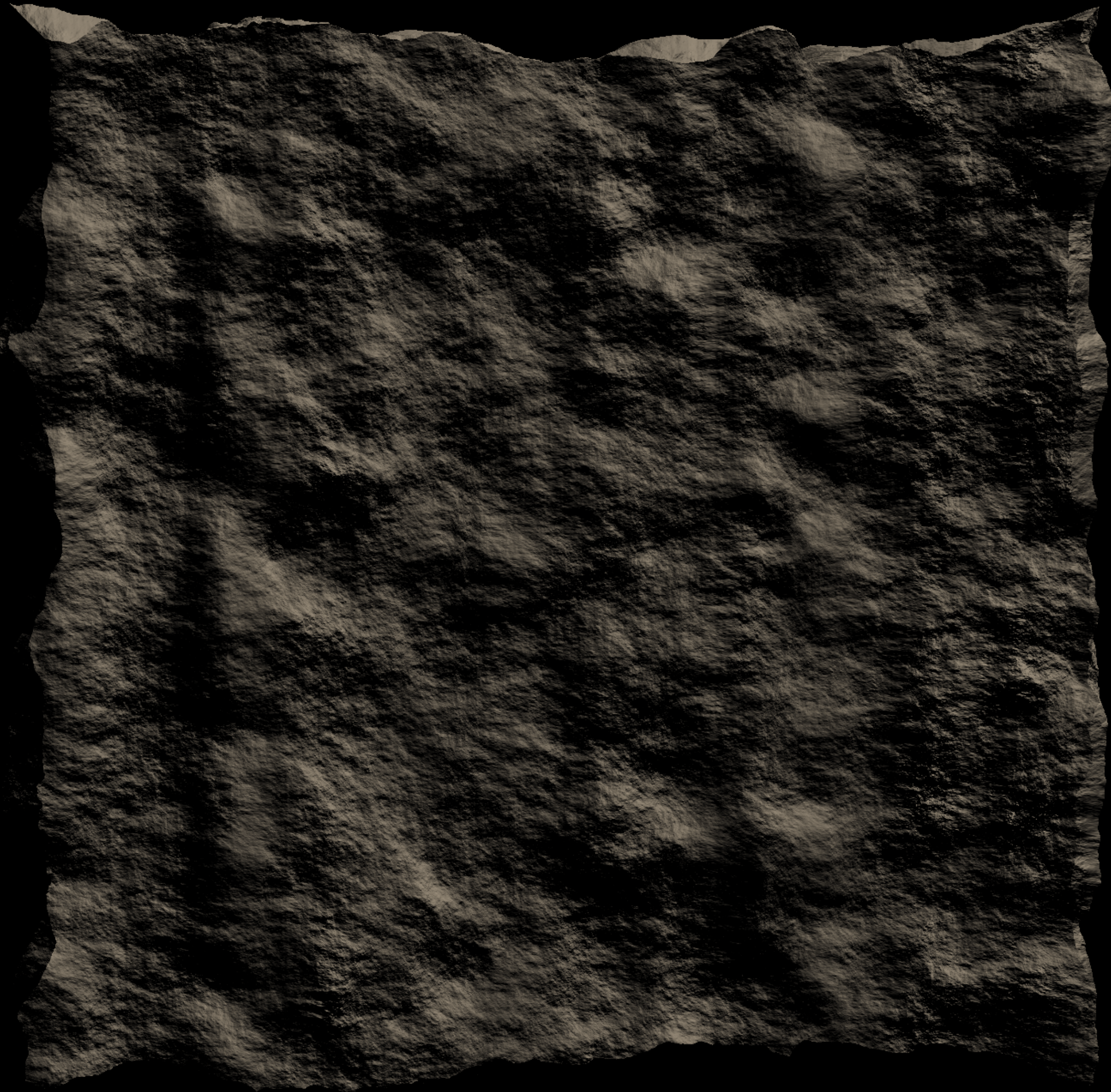
    l.x                    = mix(v.x, v.y, l.z);
    l.y                    = mix(v.z, v.w, l.z);
    return mix(l.x, l.y, l.w);
}

float fractal_brownian_motion(vec2 uv)
{
    float amplitude = .5;
    float frequency = 2.;

    float result = 0.;
    for(int i = 0; i < 8; i++)
    {
        result      += noise(uv*frequency)*amplitude;
        amplitude   *= .5;
        frequency    *= 2.;
    }

    return result;
}
```









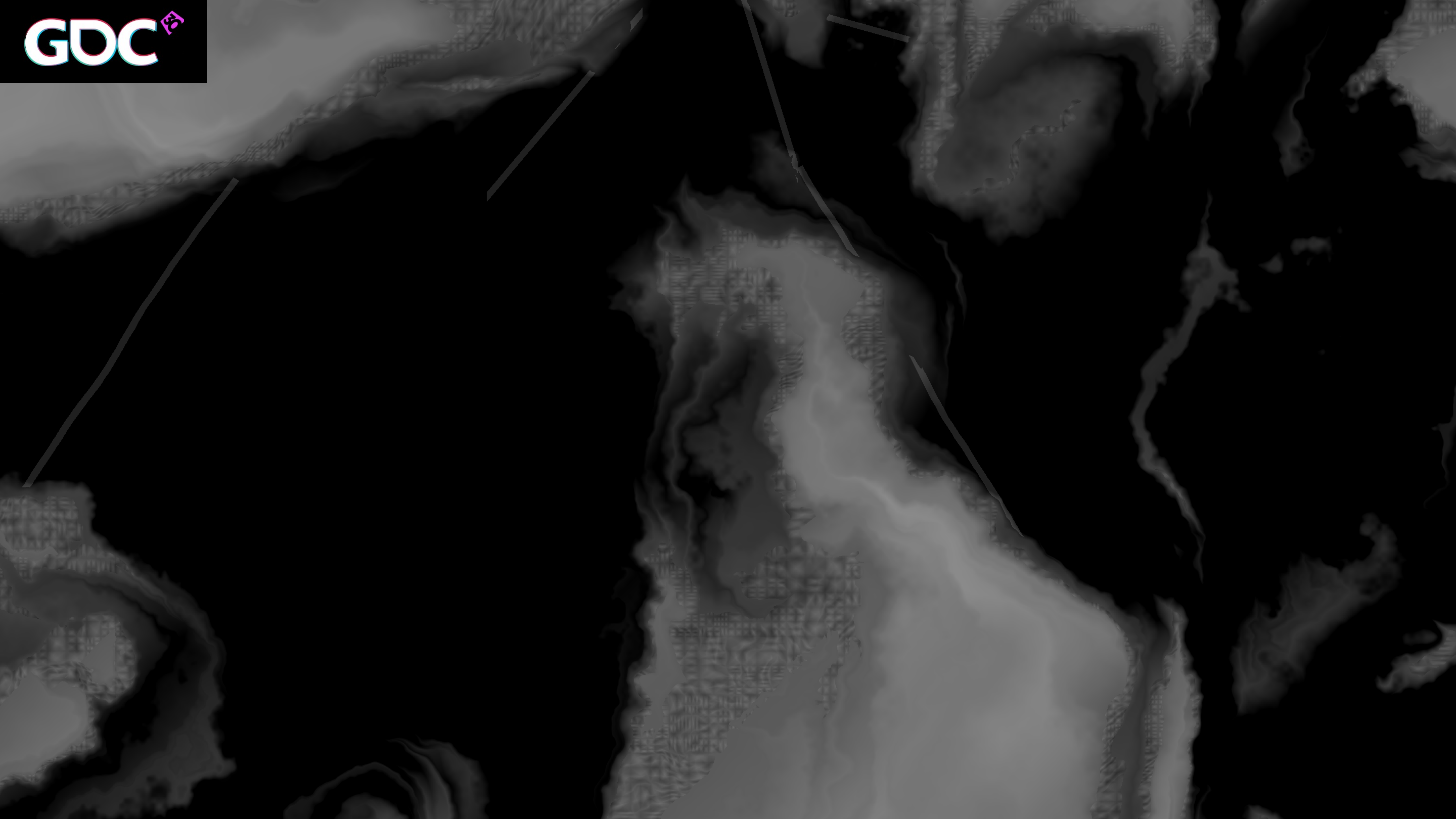




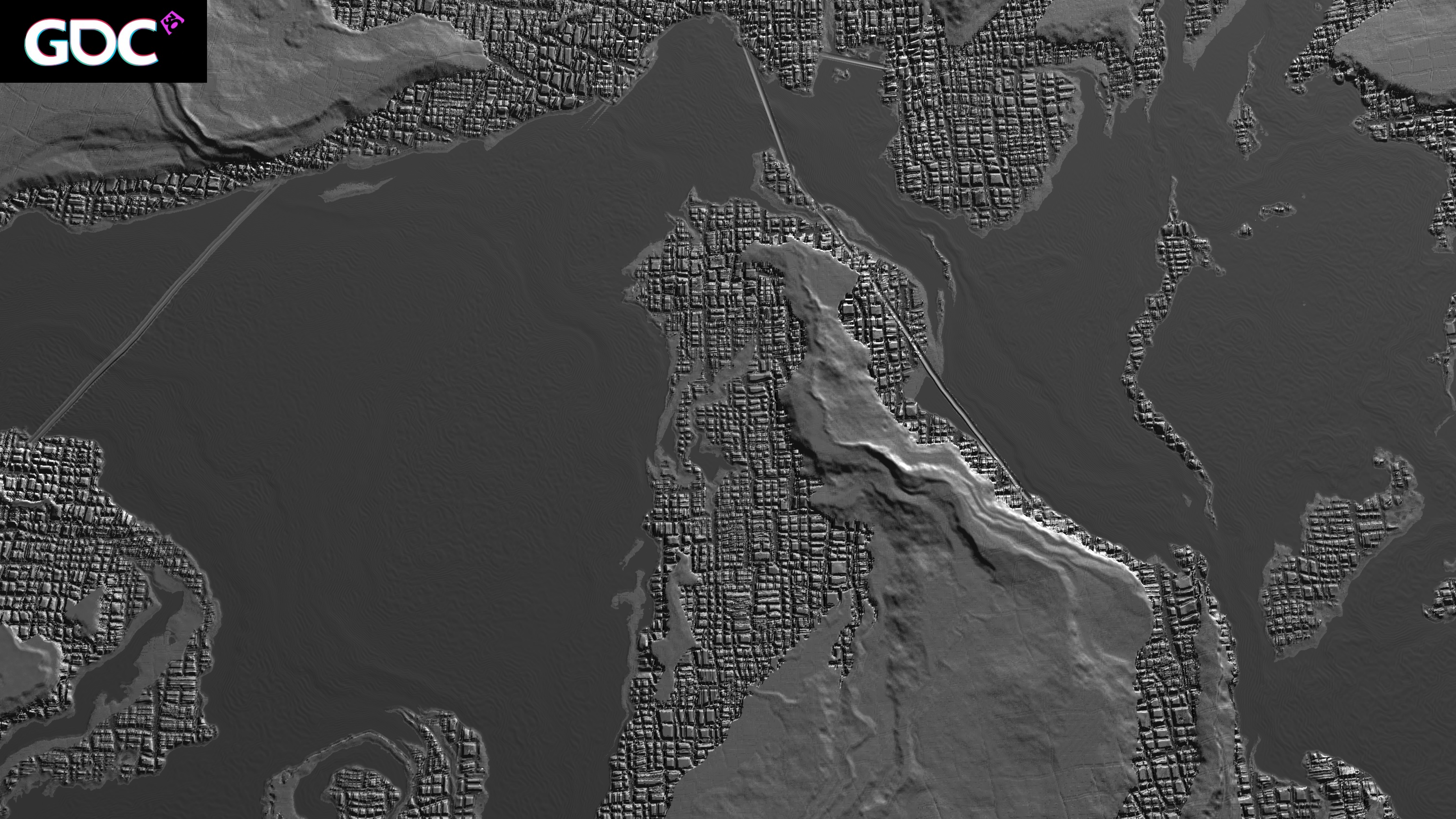




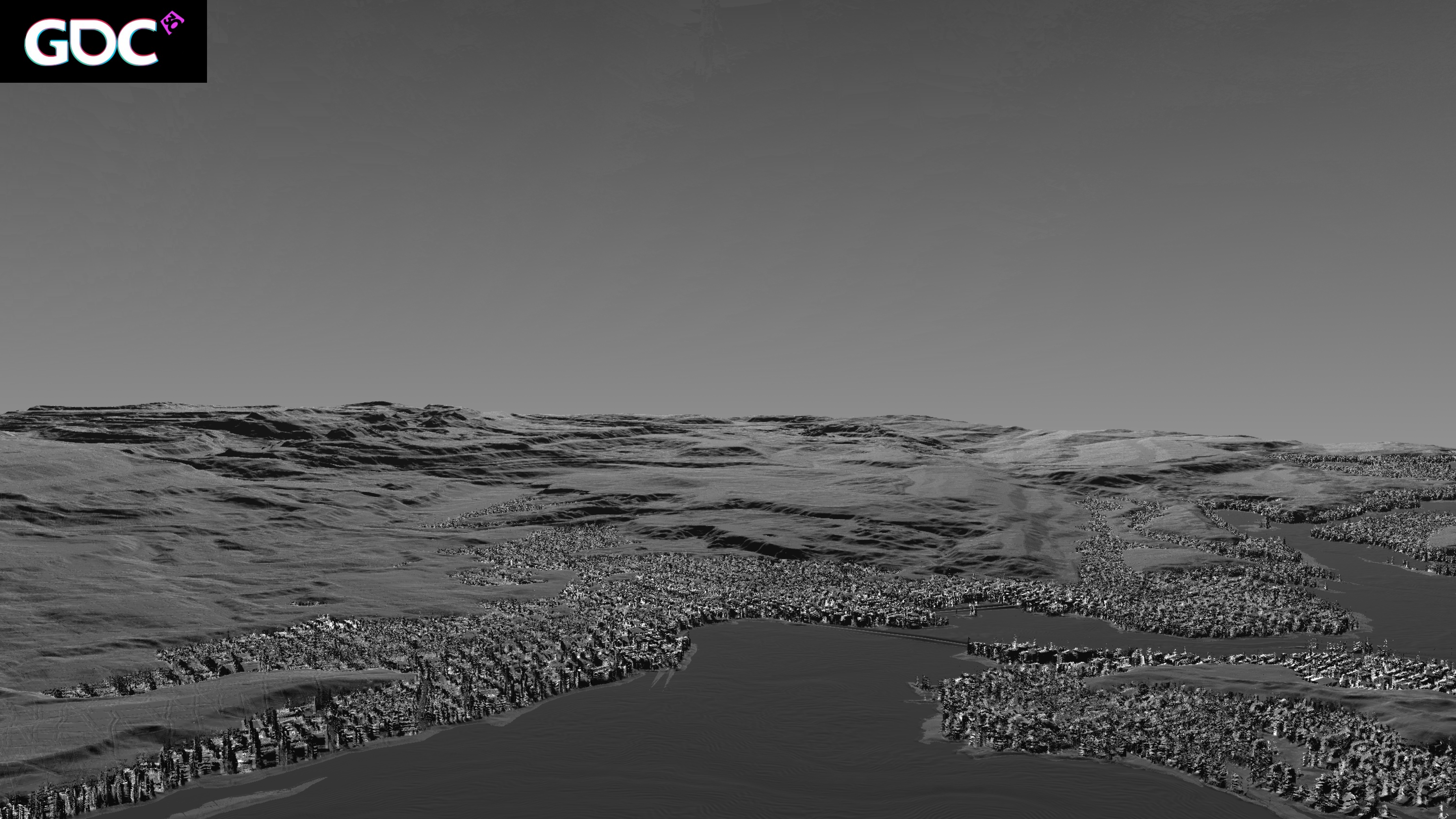












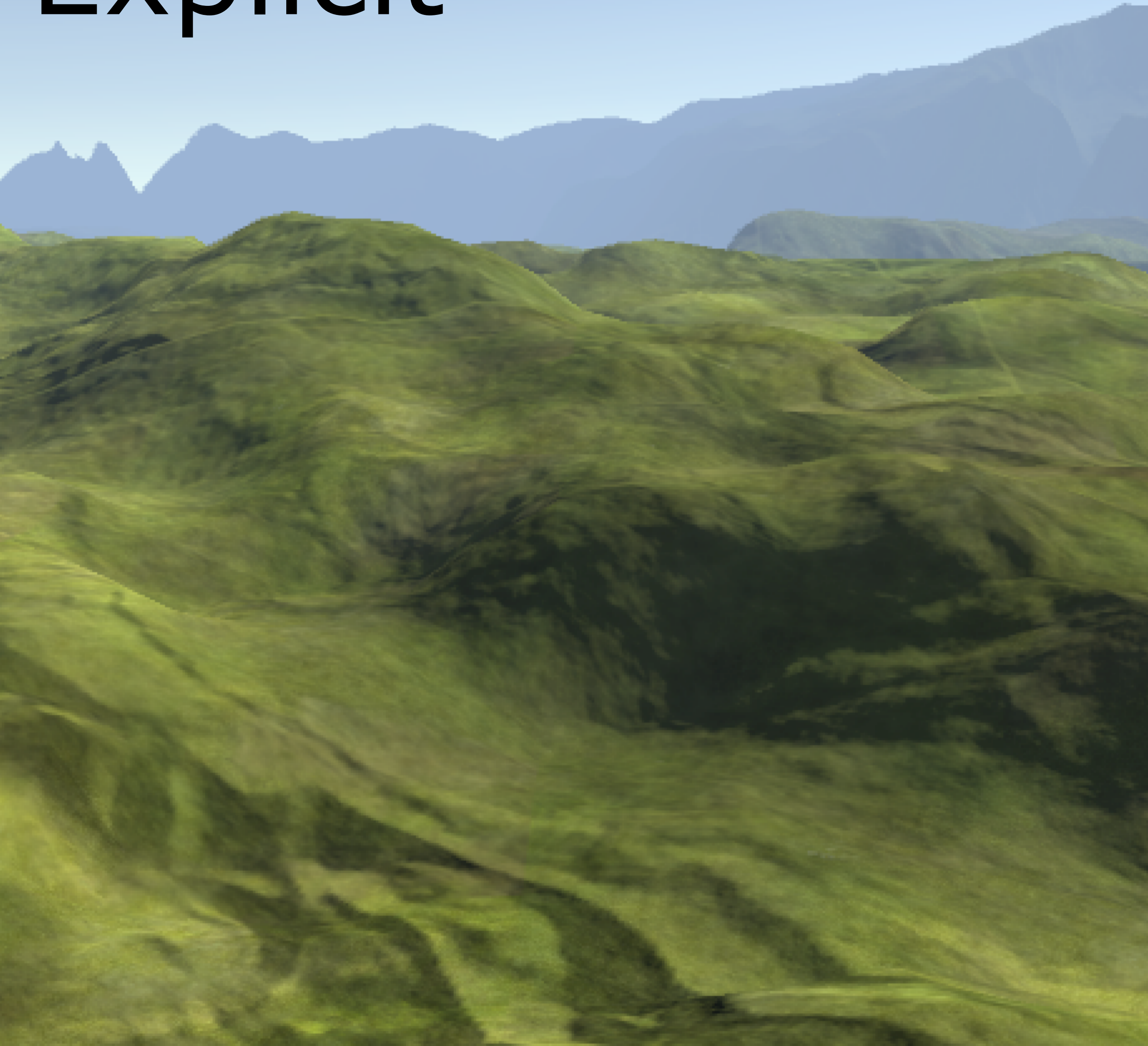




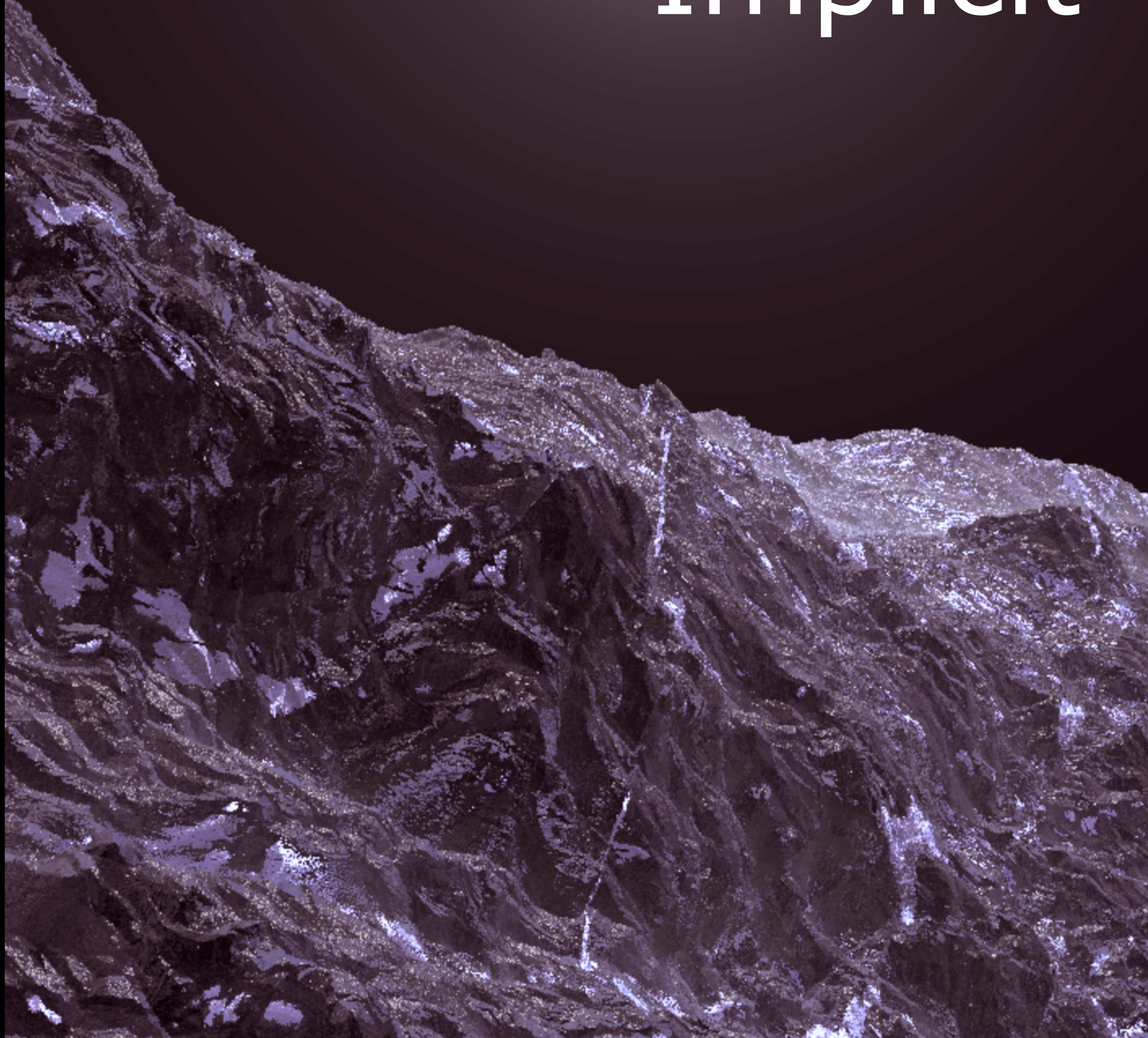
# Explicit Data vs Implicit Functions



Explicit

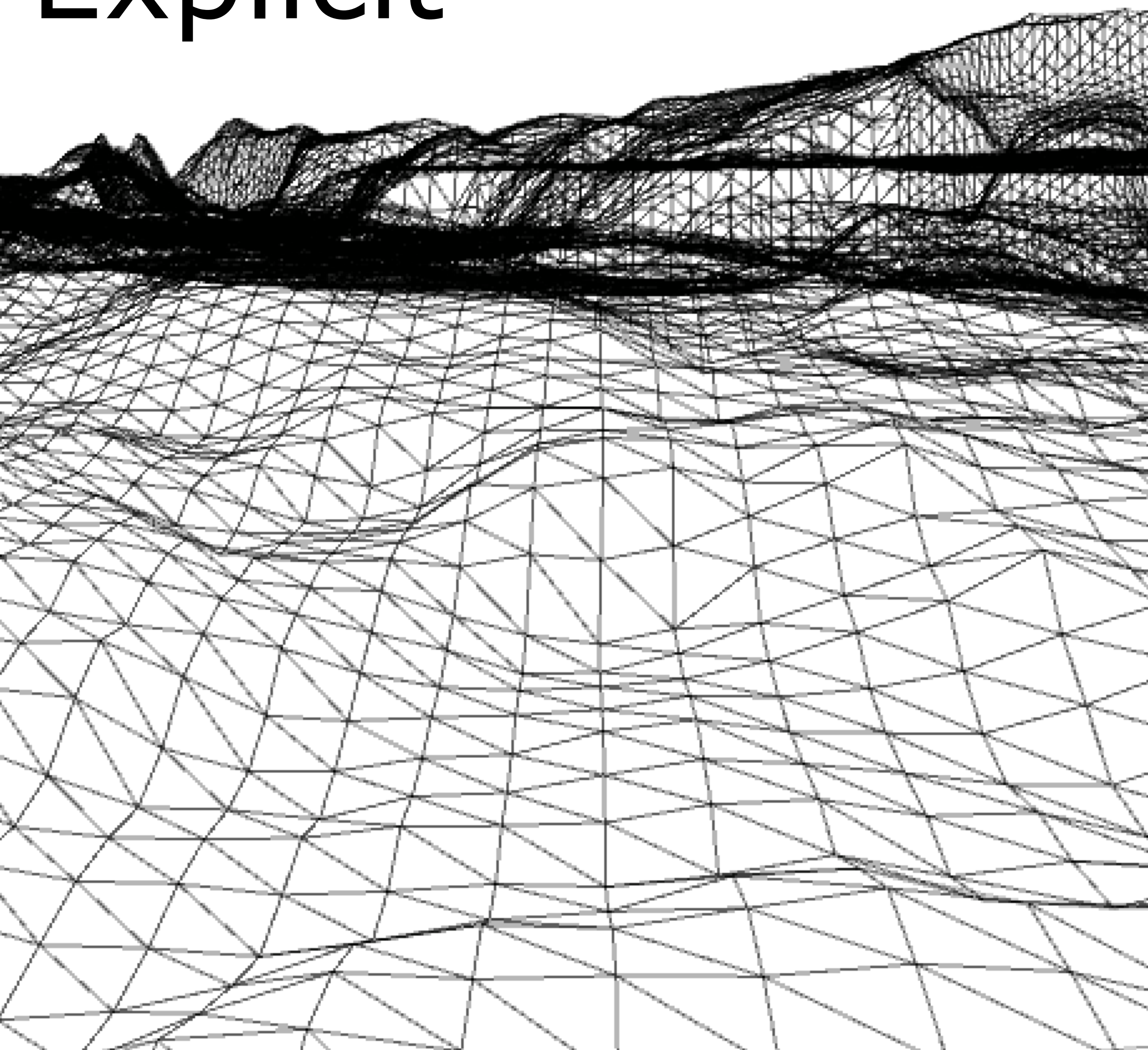


Implicit

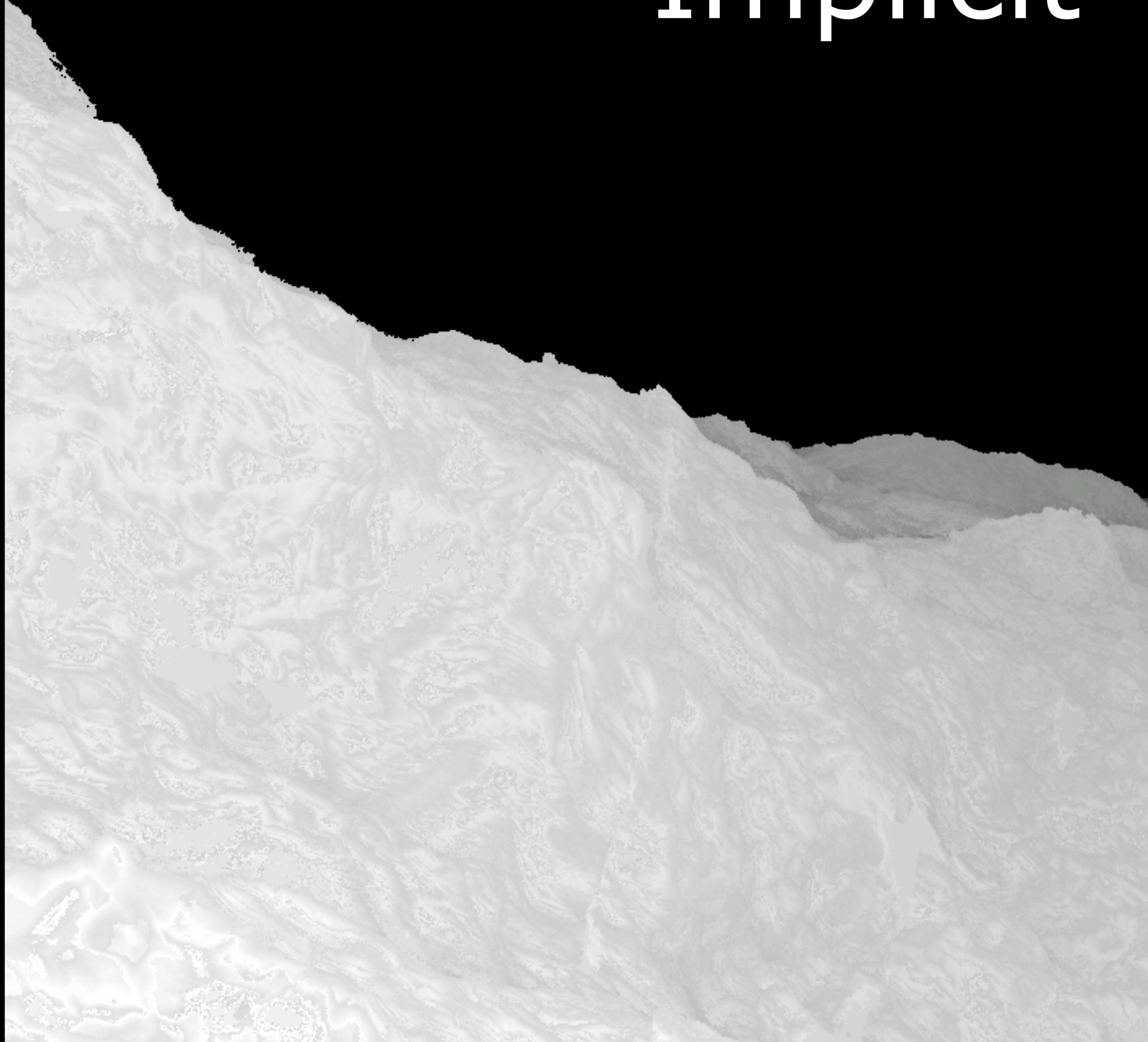




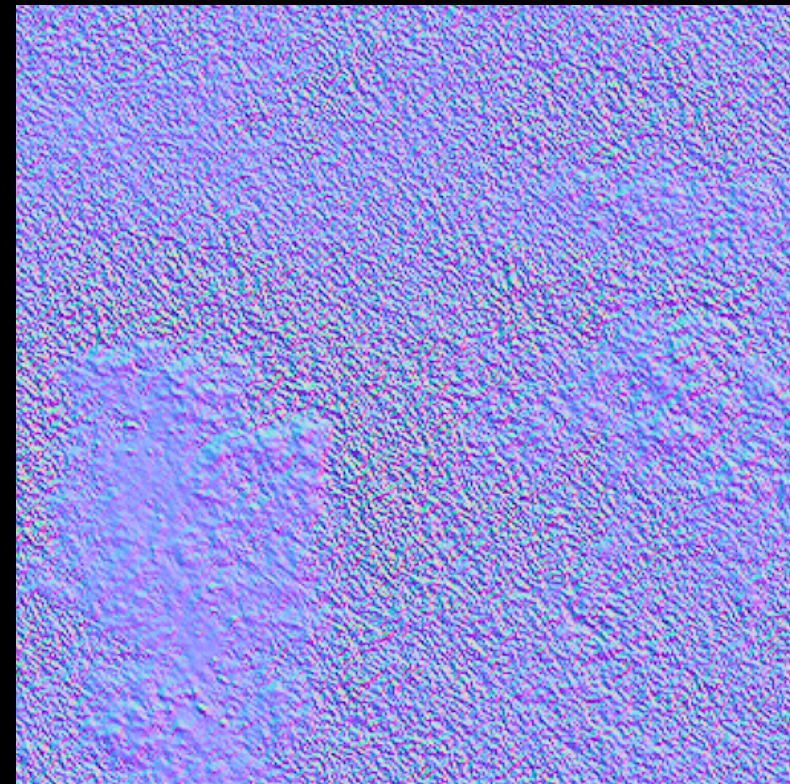
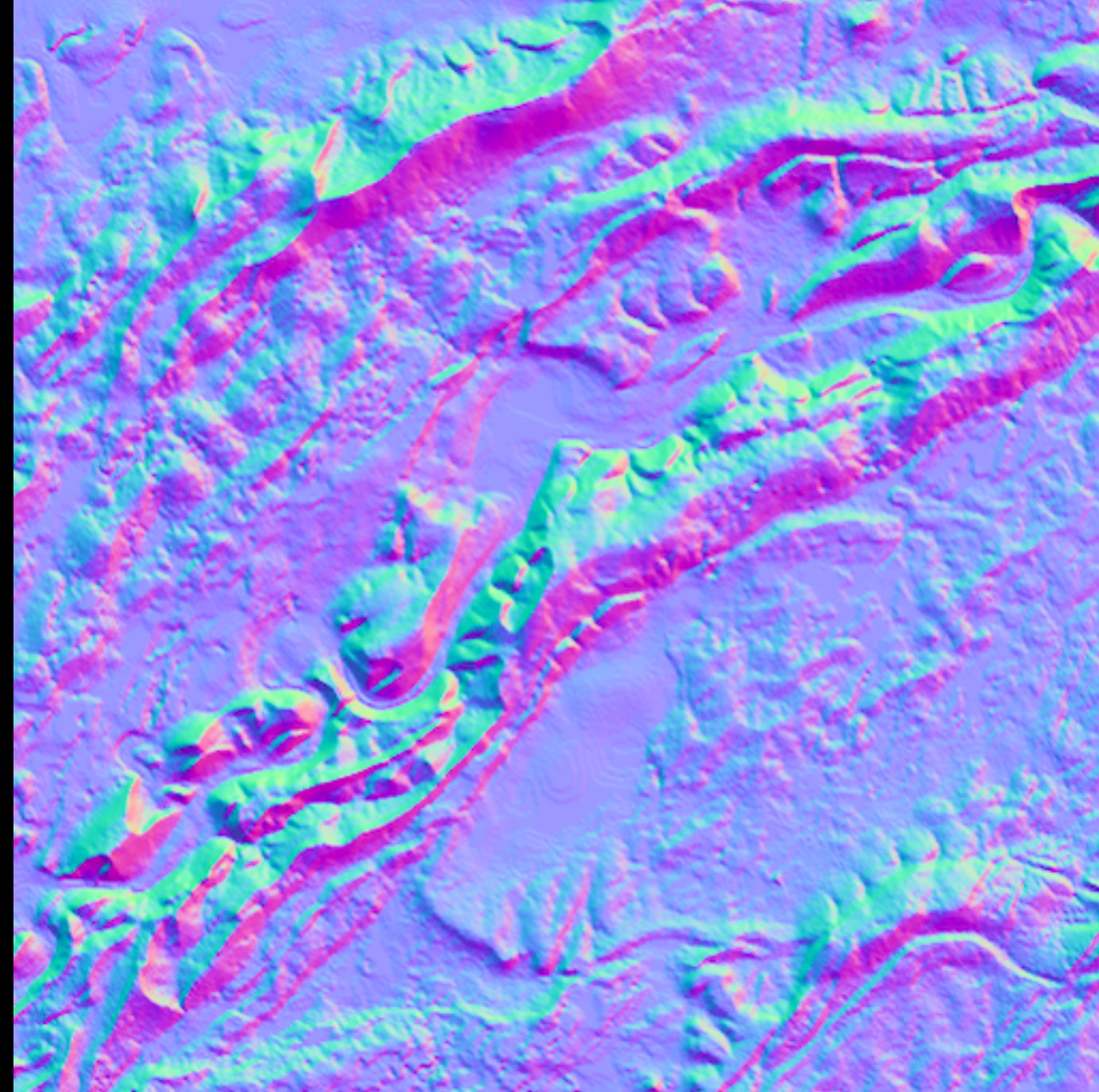
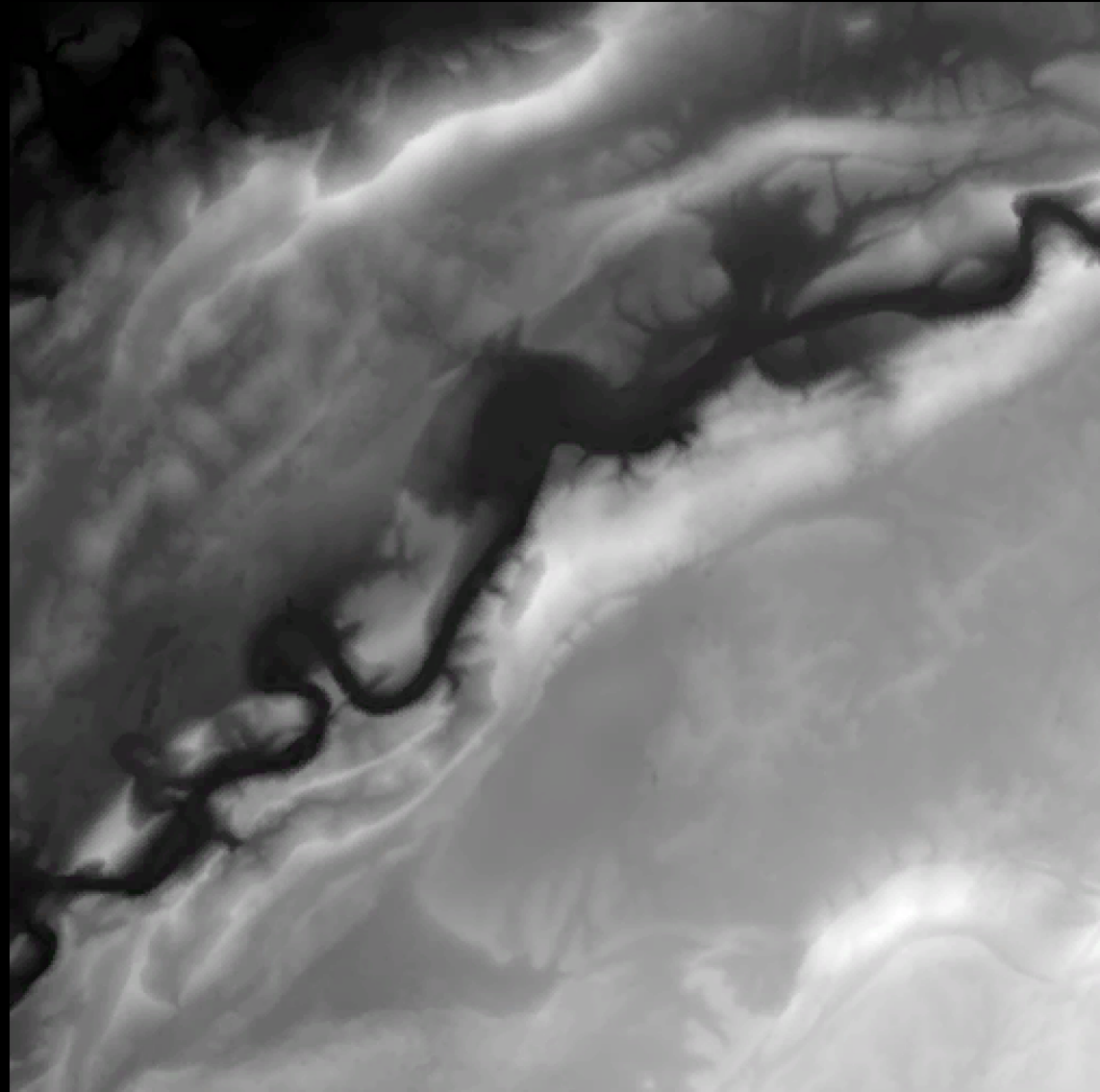
# Explicit



# Implicit







```
float fold(in float x)
{
    return abs(fract(x)-.5);
}

vec3 fold(in vec3 p)
{
    float x = fold(p.x);
    return vec3(fold(p.z+fold(p.y)), fold(p.z+x), fold(p.y+x));
}

float fold_harmonic(in vec3 p, const int it)
{
    vec3 bp = abs(p);

    float rz = 1.;
    for (int i = 0; i < 32; i++)
    {
        if(i>it) break;
        p      *= fold(bp);
        bp     += abs(fract(bp))-rz;
        rz     += dot(fold(p), bp)-rz;

    }
    return rz*float(it);
}
```





# Explicit Data





# Explicit Data

Data is stored in memory as independent values.





# Explicit Data

Data is stored in memory as independent values.

Mesh vertices, texture pixels, etc...





# Explicit Data

Data is stored in memory as independent values.

Mesh vertices, texture pixels, etc...

Data is read from memory.





# Implicit Functions





# Implicit Functions

Code is data.





# Implicit Functions

Code is data.

Everything is procedural.





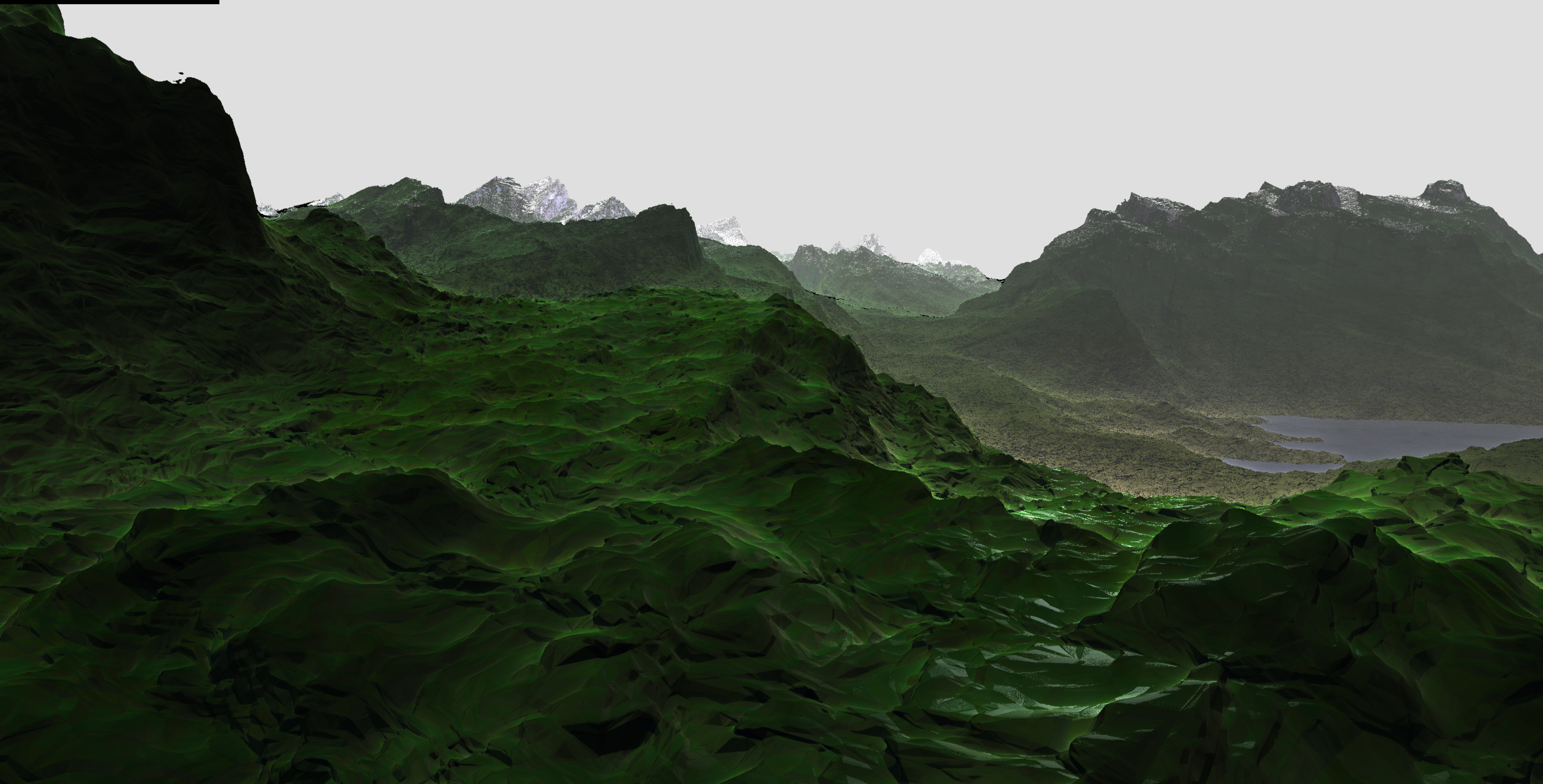
# Implicit Functions

Code is data.

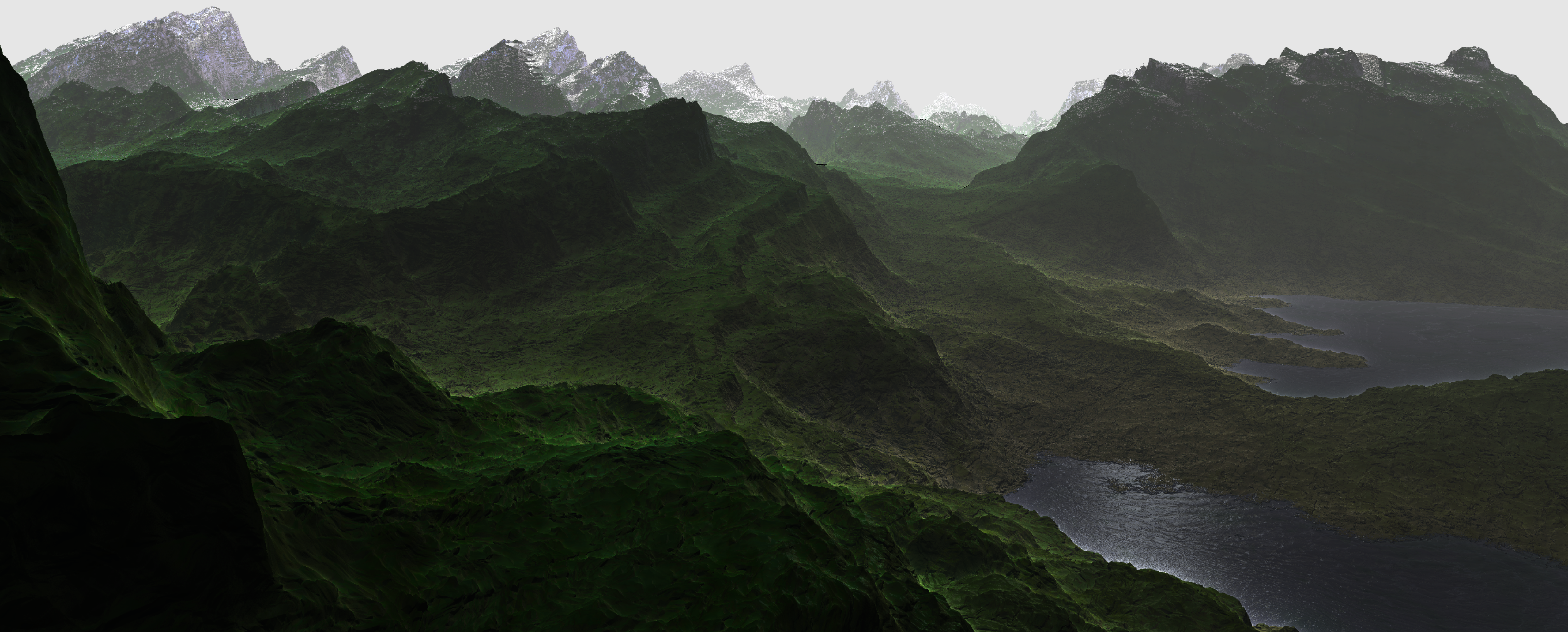
Everything is procedural.

Data is accessed via computation.













# Sphere Tracing





# Sphere Tracing

Sphere tracing is a form of raytracing  
(there are many.)





# Sphere Tracing

Sphere tracing is a form of raytracing  
(there are many.)

It is ideal for implicit functions.



# Sphere Tracing

Sphere tracing is a form of raytracing  
(there are many.)

It is ideal for implicit functions.

It is not a replacement for rasterization or voxels.





# Sphere Tracing

It is inefficient.





# Sphere Tracing

It is inefficient.

It is simple.





# Sphere Tracing

It is inefficient.

It is simple.

It is powerful.





Construct the view.

Trace the ray.

Determine which way the surface is facing.

Add light.





# Sphere Tracing

## 4 Steps





Construct the view.

# Sphere Tracing

## 4 Steps





# Sphere Tracing

## 4 Steps

Construct the view.

Trace the ray.





# Sphere Tracing

## 4 Steps

Construct the view.

Trace the ray.

Determine which way the surface is facing.





# Sphere Tracing

## 4 Steps

Construct the view.

Trace the ray.

Determine which way the surface is facing.

Add light.





Construct the view.

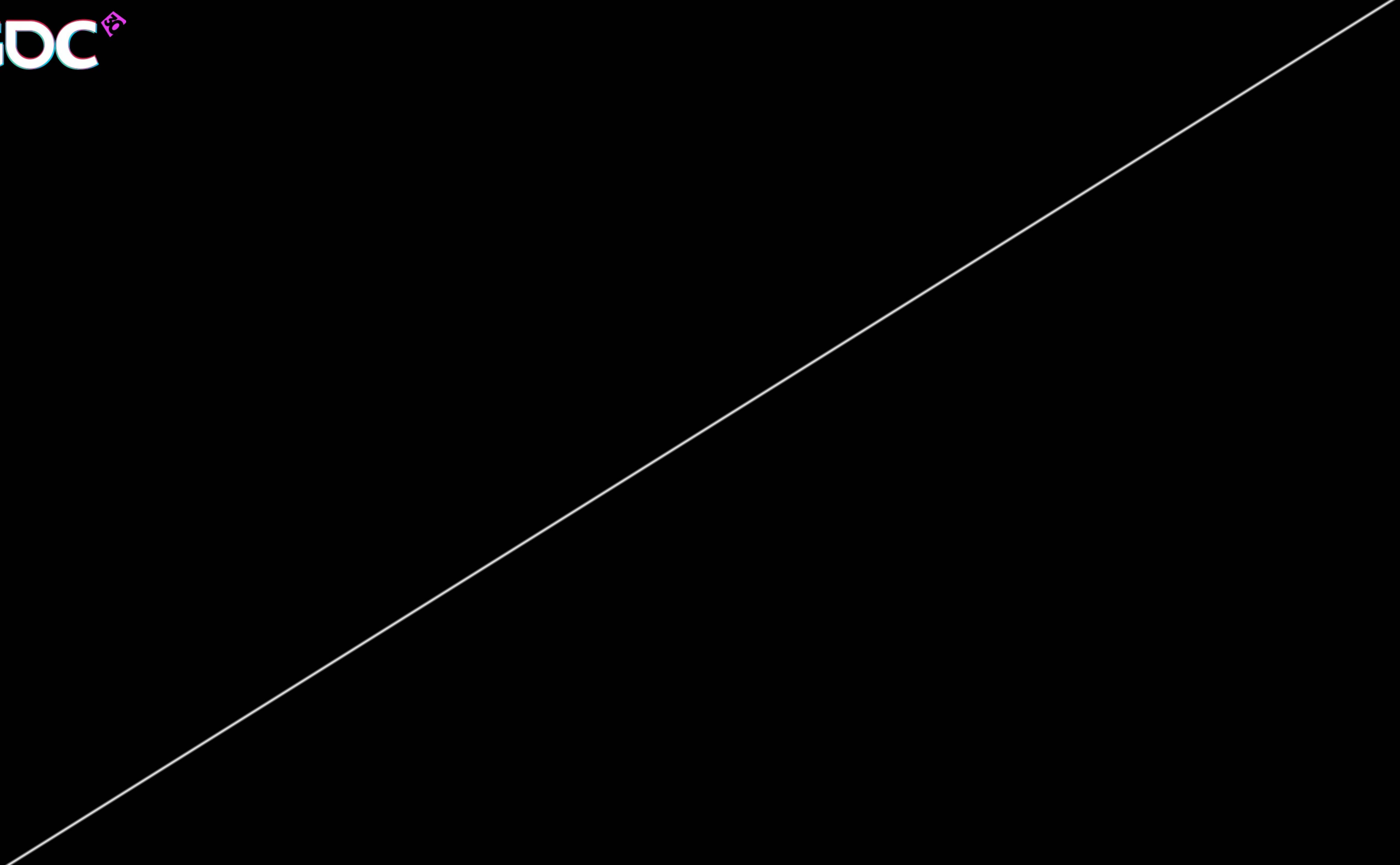
Trace the ray.

---

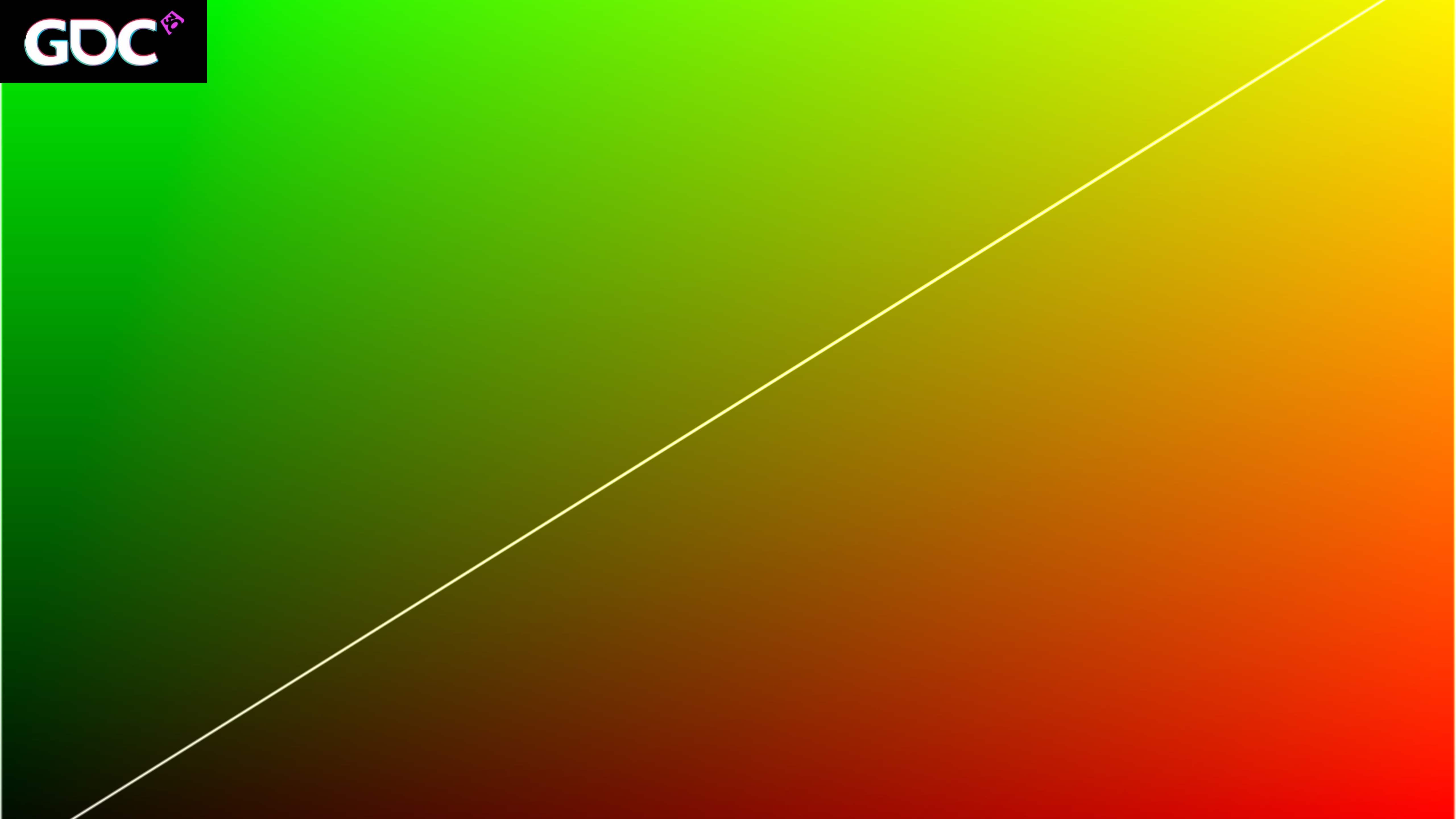
Determine which way the surface is facing.

Add light.









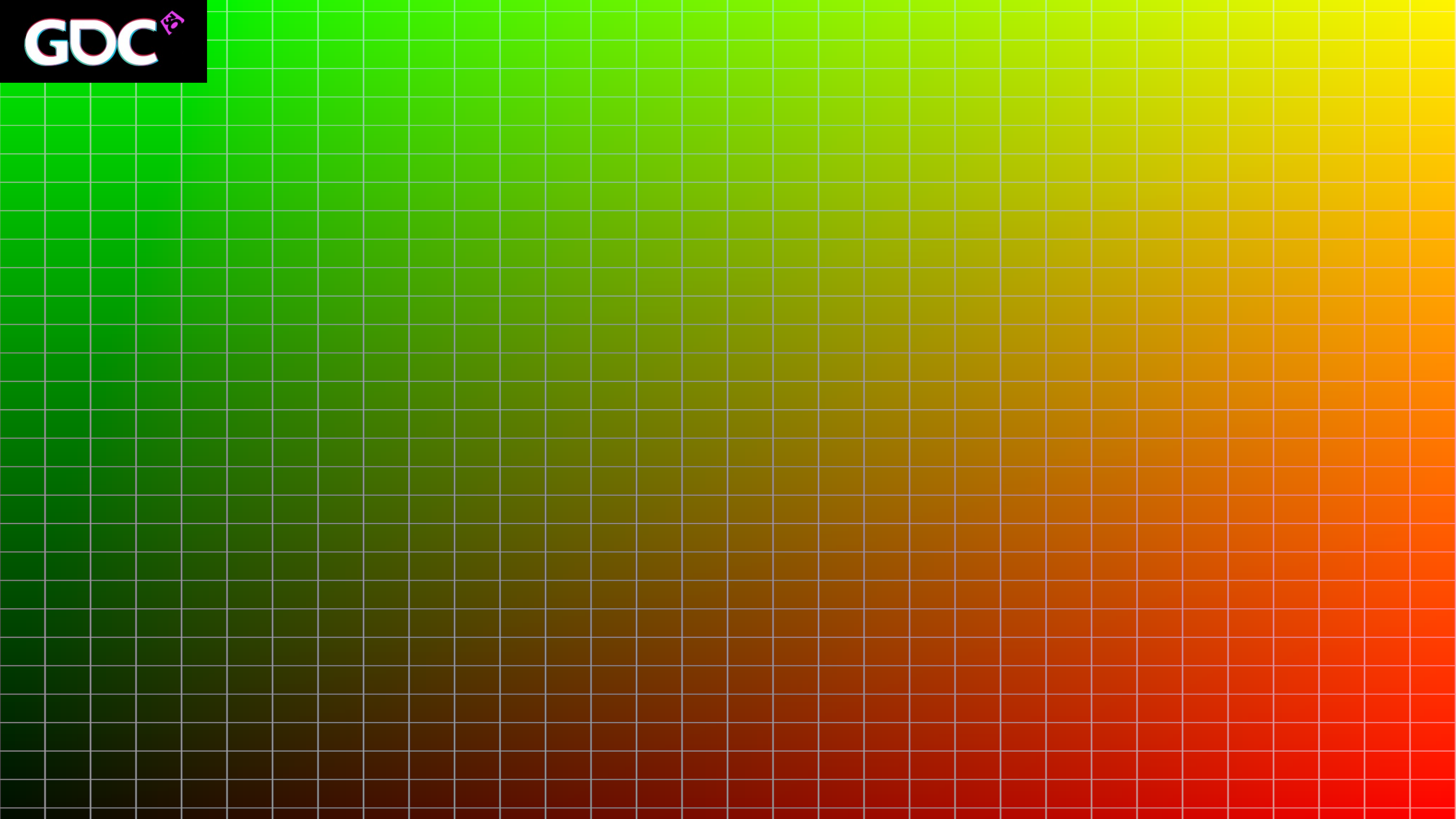














```
vec2 screen_coordinates = gl_FragCoord.xy;  
screen_coordinates /= resolution;  
screen_coordinates = screen_coordinates - .5;  
screen_coordinates *= resolution/min(resolution.x, resolution.y);  
  
float field_of_view = 1.5;  
  
vec3 direction = vec3(screen_coordinates, field_of_view);  
direction = normalize(direction);
```



```
vec2 screen_coordinates = gl_FragCoord.xy;  
screen_coordinates /= resolution;  
screen_coordinates = screen_coordinates - .5;  
screen_coordinates *= resolution/min(resolution.x, resolution.y);  
  
float field_of_view = 1.5;  
  
vec3 direction = vec3(screen_coordinates, field_of_view);  
direction = normalize(direction);
```



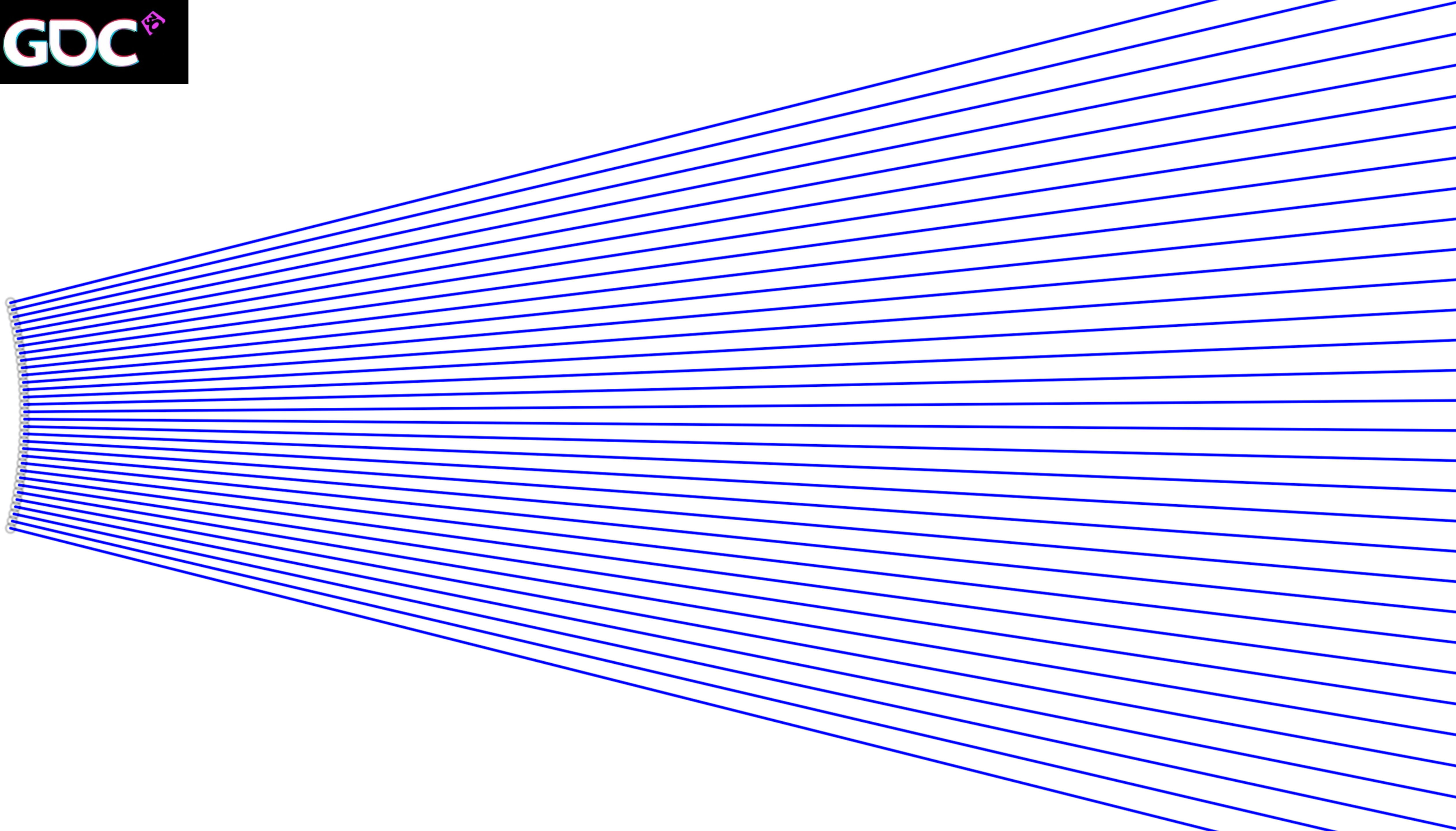
```
vec2 screen_coordinates = gl_FragCoord.xy;  
screen_coordinates /= resolution;  
screen_coordinates = screen_coordinates - .5;  
screen_coordinates *= resolution/min(resolution.x, resolution.y);  
  
float field_of_view = 1.5;  
  
vec3 direction = vec3(screen_coordinates, field_of_view);  
direction = normalize(direction);
```





A series of 30 horizontal blue lines spanning the width of the page, intended for handwritten notes. Each line is preceded by a small, light gray circular dot on the left margin, serving as a bullet point or guide for line placement.





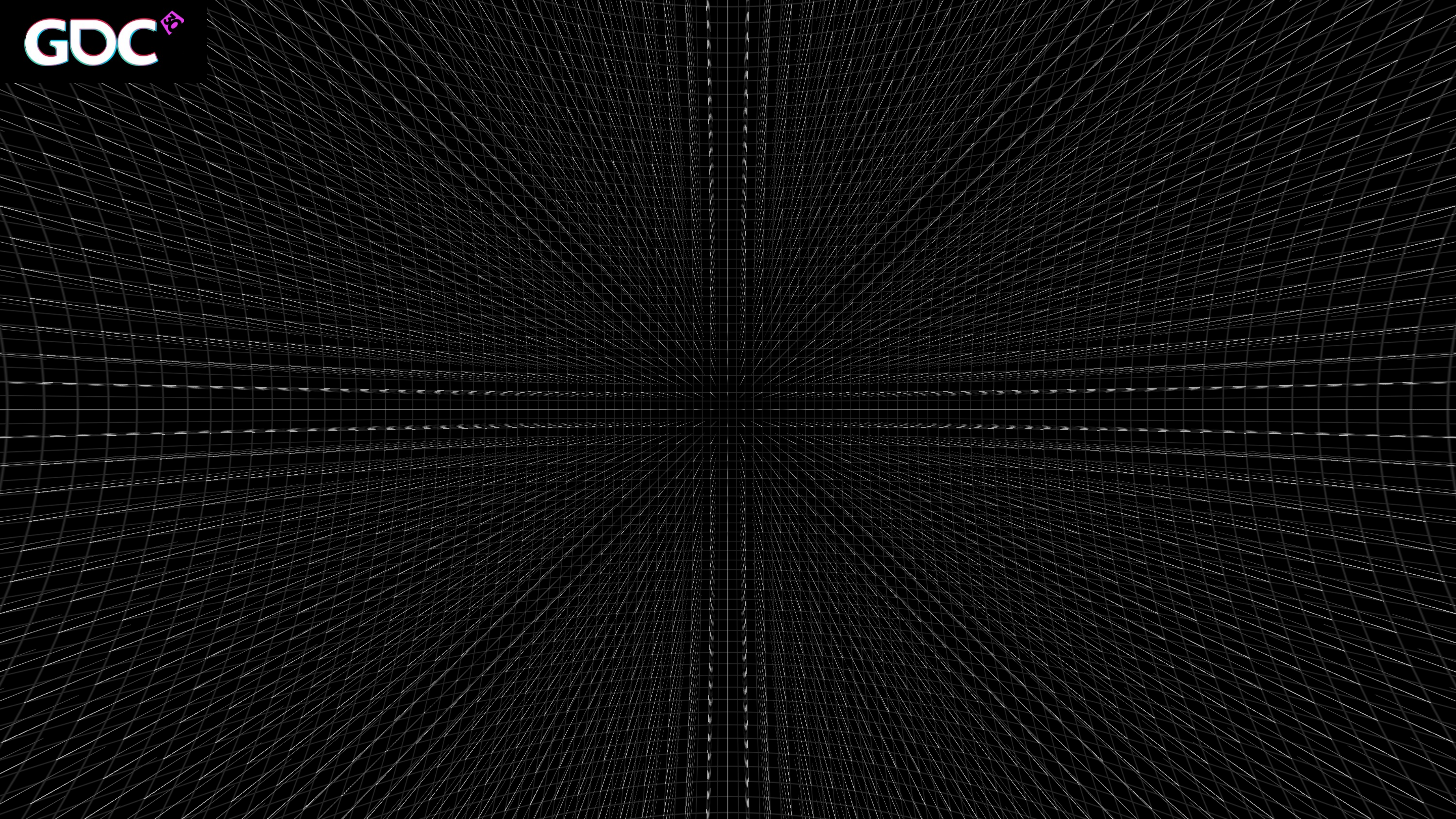


```
vec2 screen_coordinates = gl_FragCoord.xy;  
screen_coordinates /= resolution;  
screen_coordinates = screen_coordinates - .5;  
screen_coordinates *= resolution/min(resolution.x, resolution.y);  
  
float field_of_view = 1.5;  
  
vec3 direction = vec3(screen_coordinates, field_of_view);  
direction = normalize(direction);
```

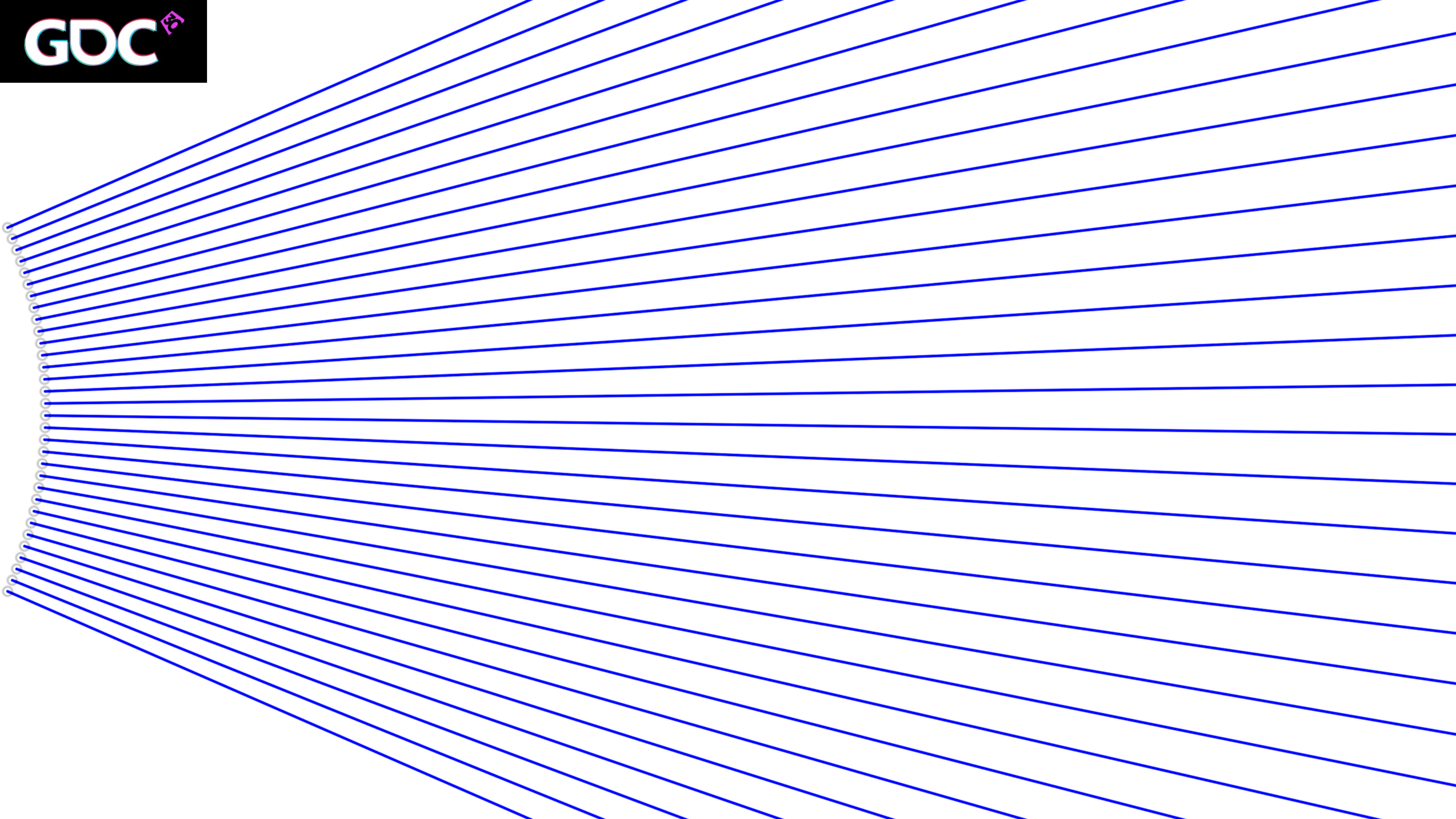


```
vec2 screen_coordinates = gl_FragCoord.xy;  
screen_coordinates /= resolution;  
screen_coordinates = screen_coordinates - .5;  
screen_coordinates *= resolution/min(resolution.x, resolution.y);  
  
float field_of_view = 1.5;  
  
vec3 direction = vec3(screen_coordinates, field_of_view);  
direction = normalize(direction);
```

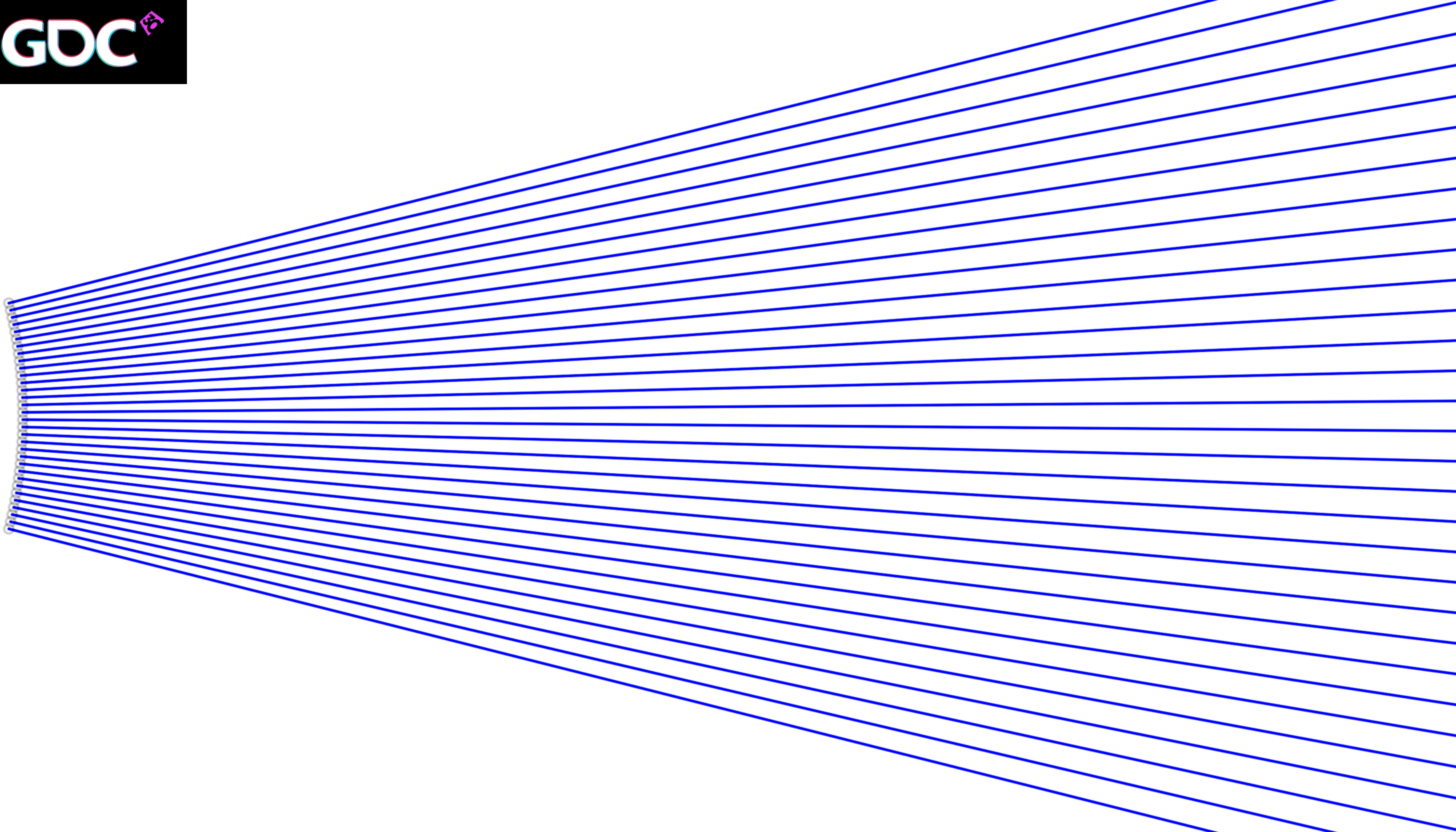




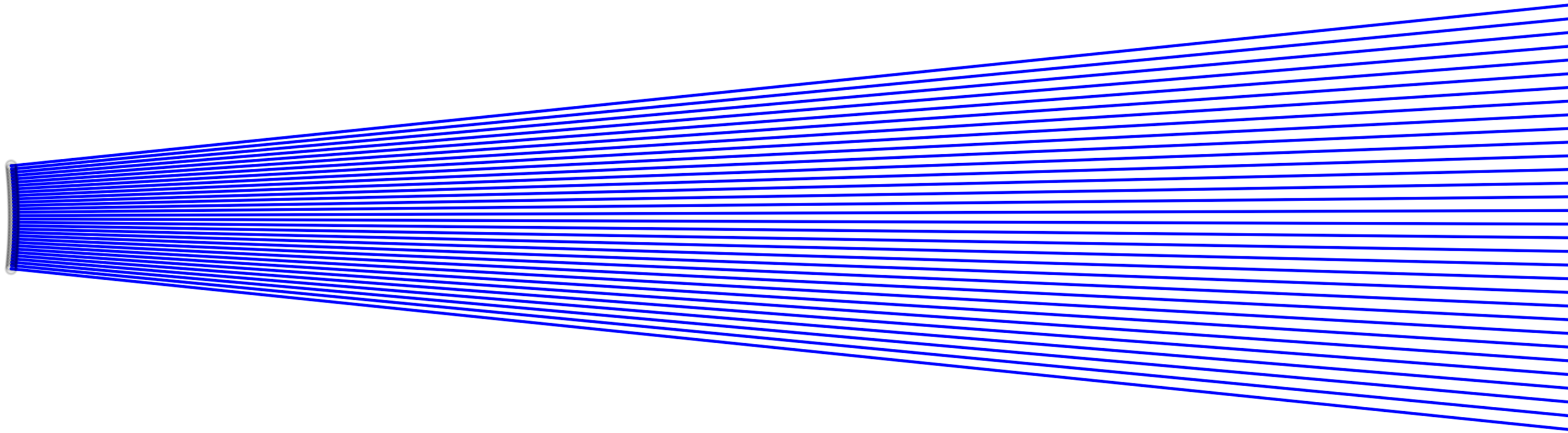




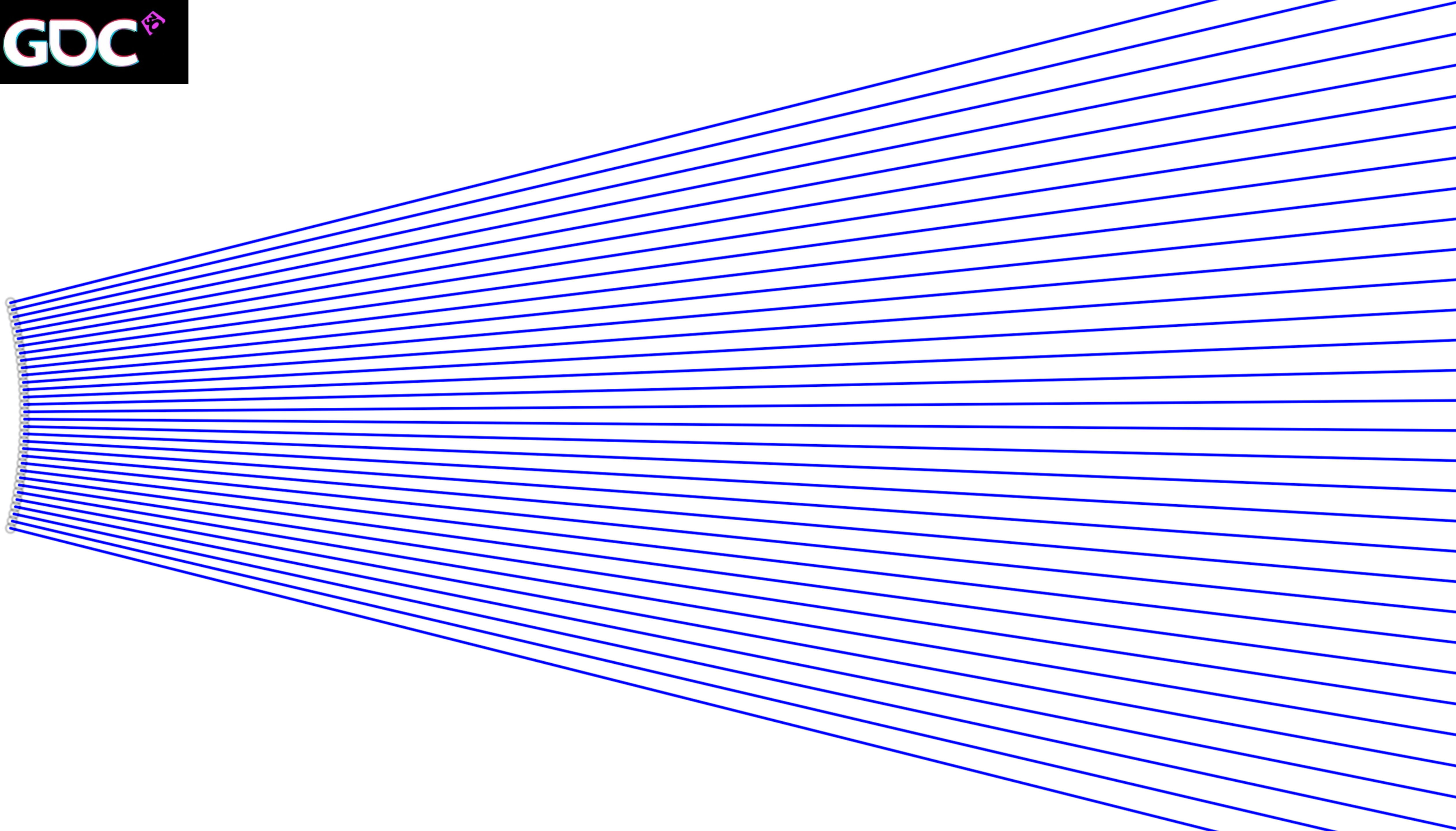




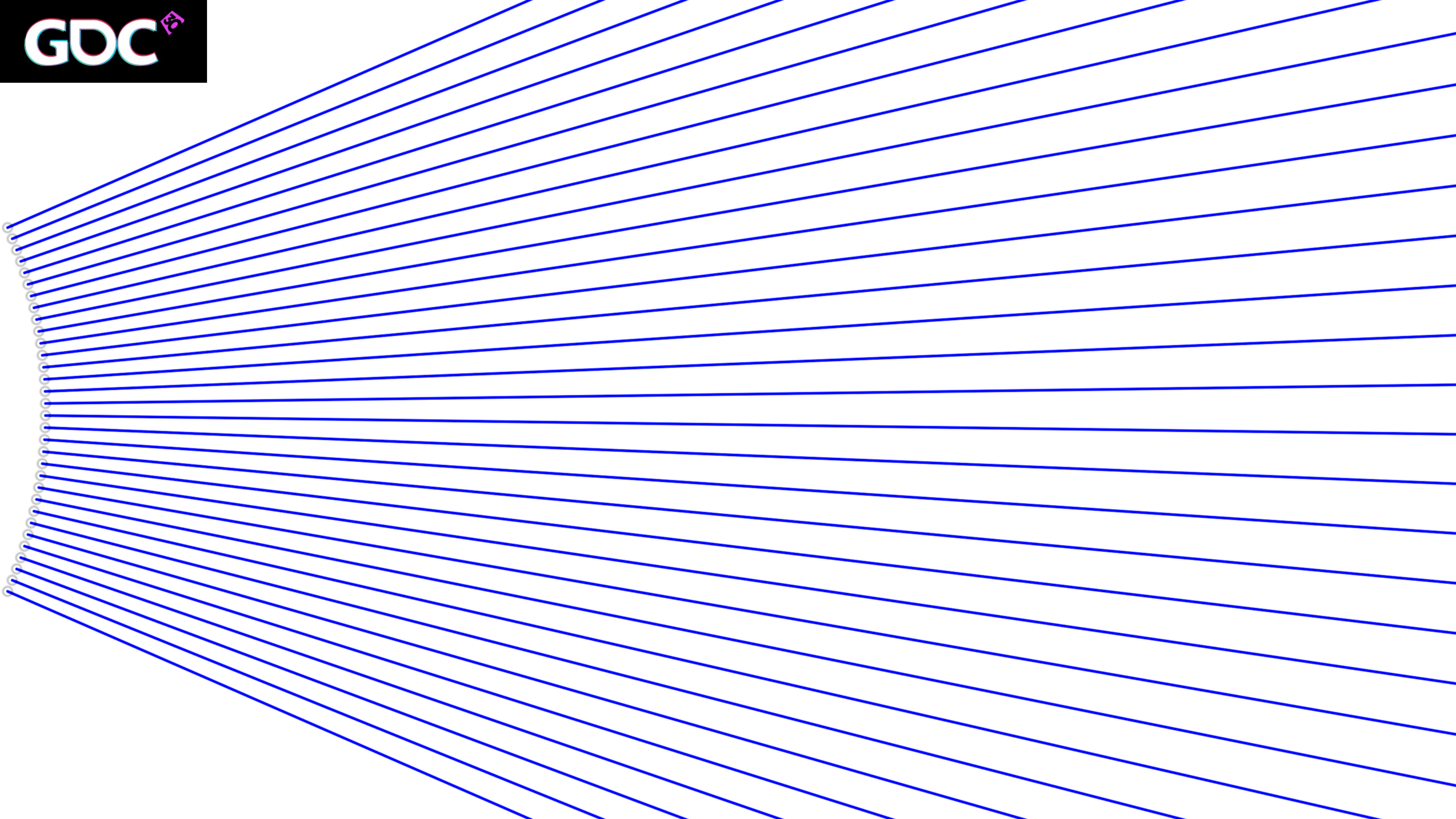














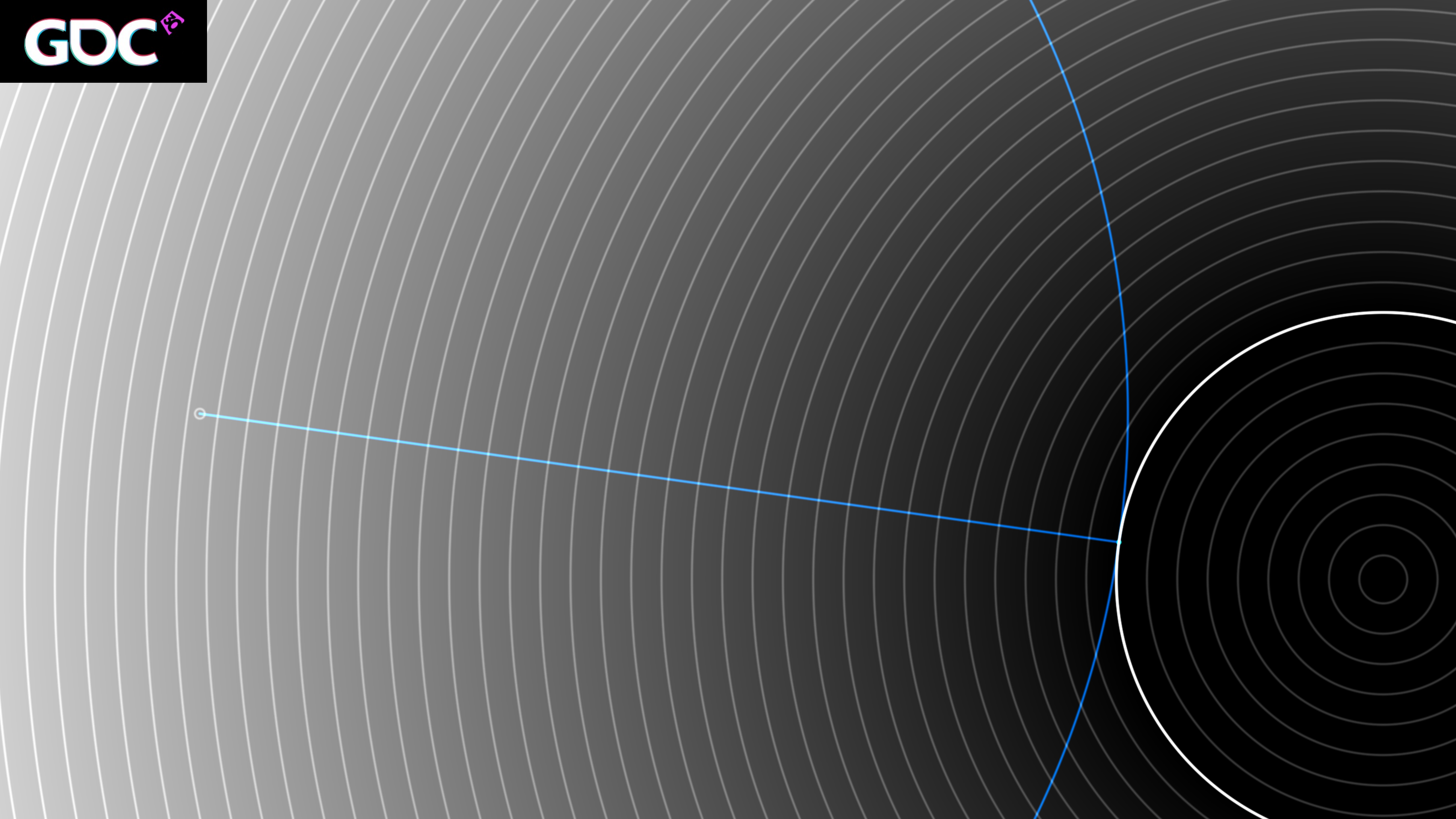




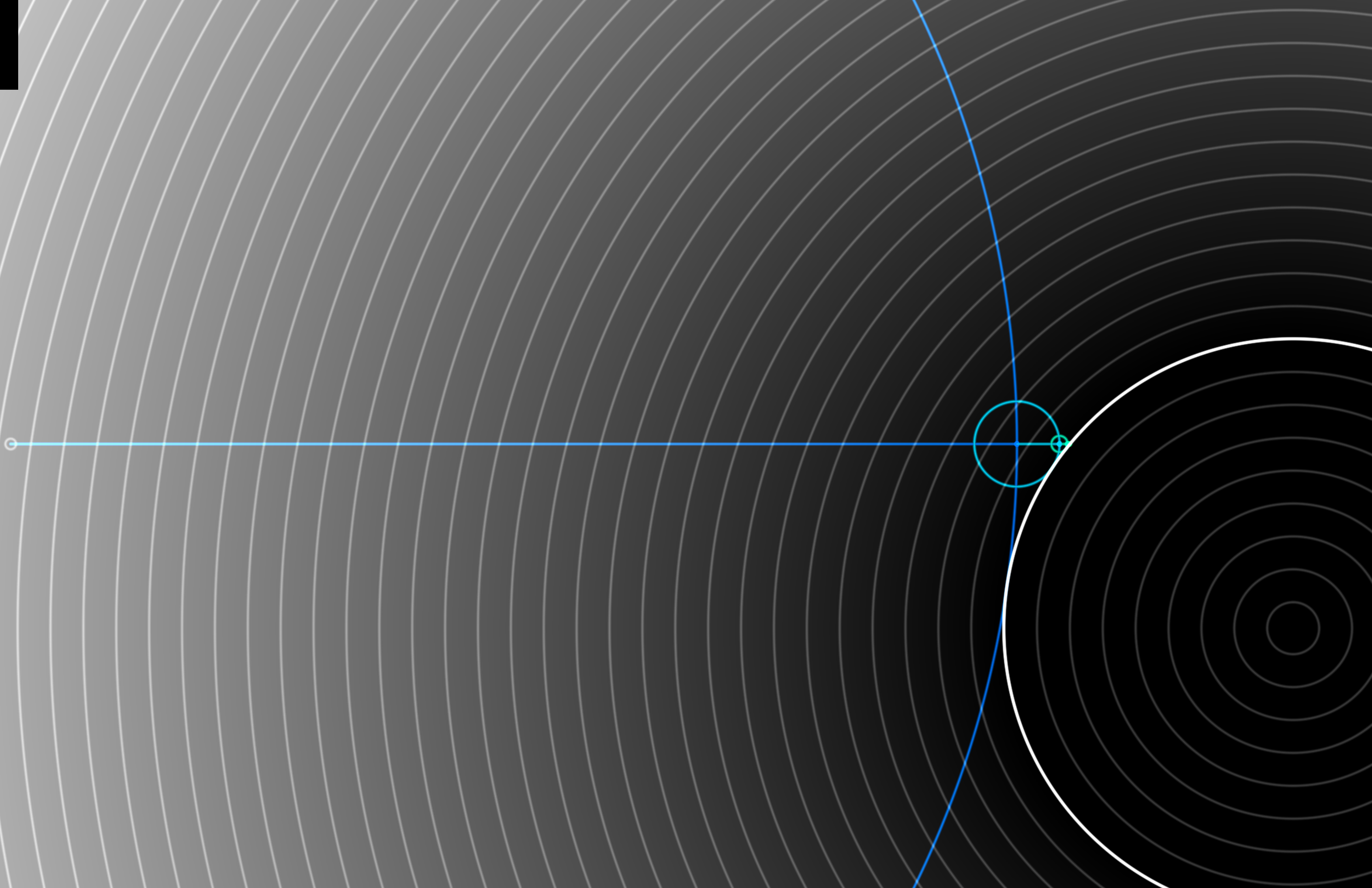
# Rays



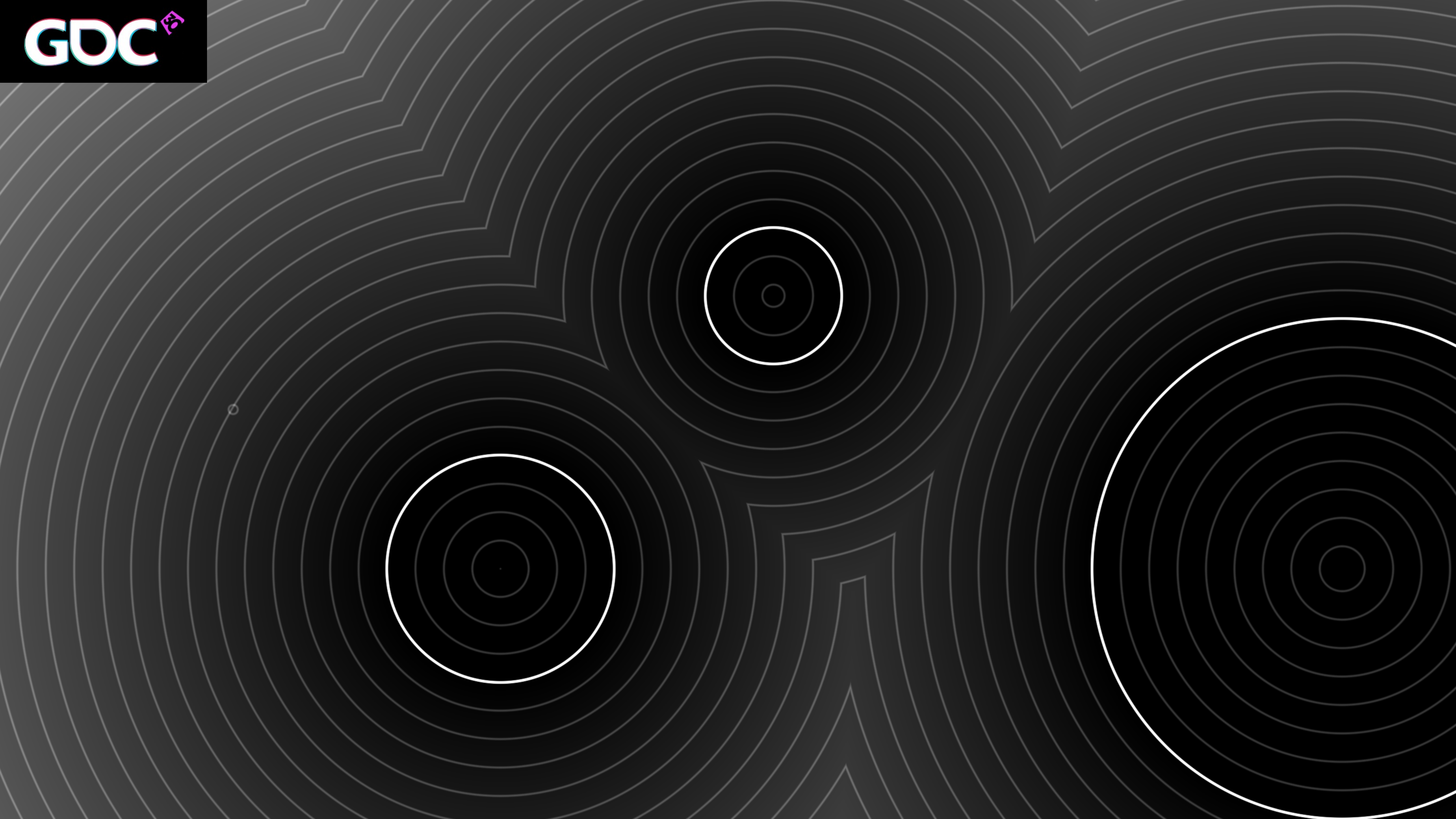




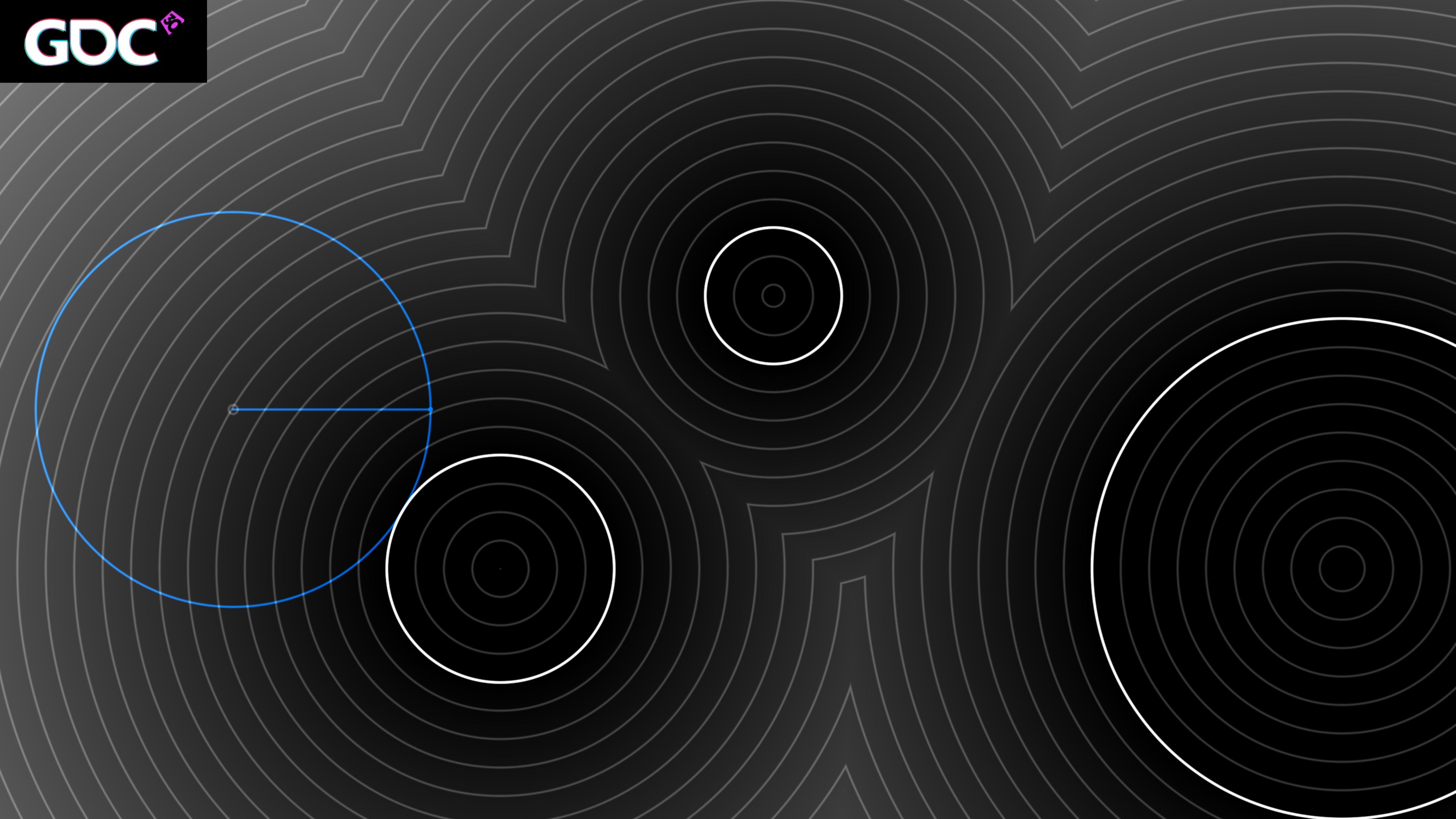




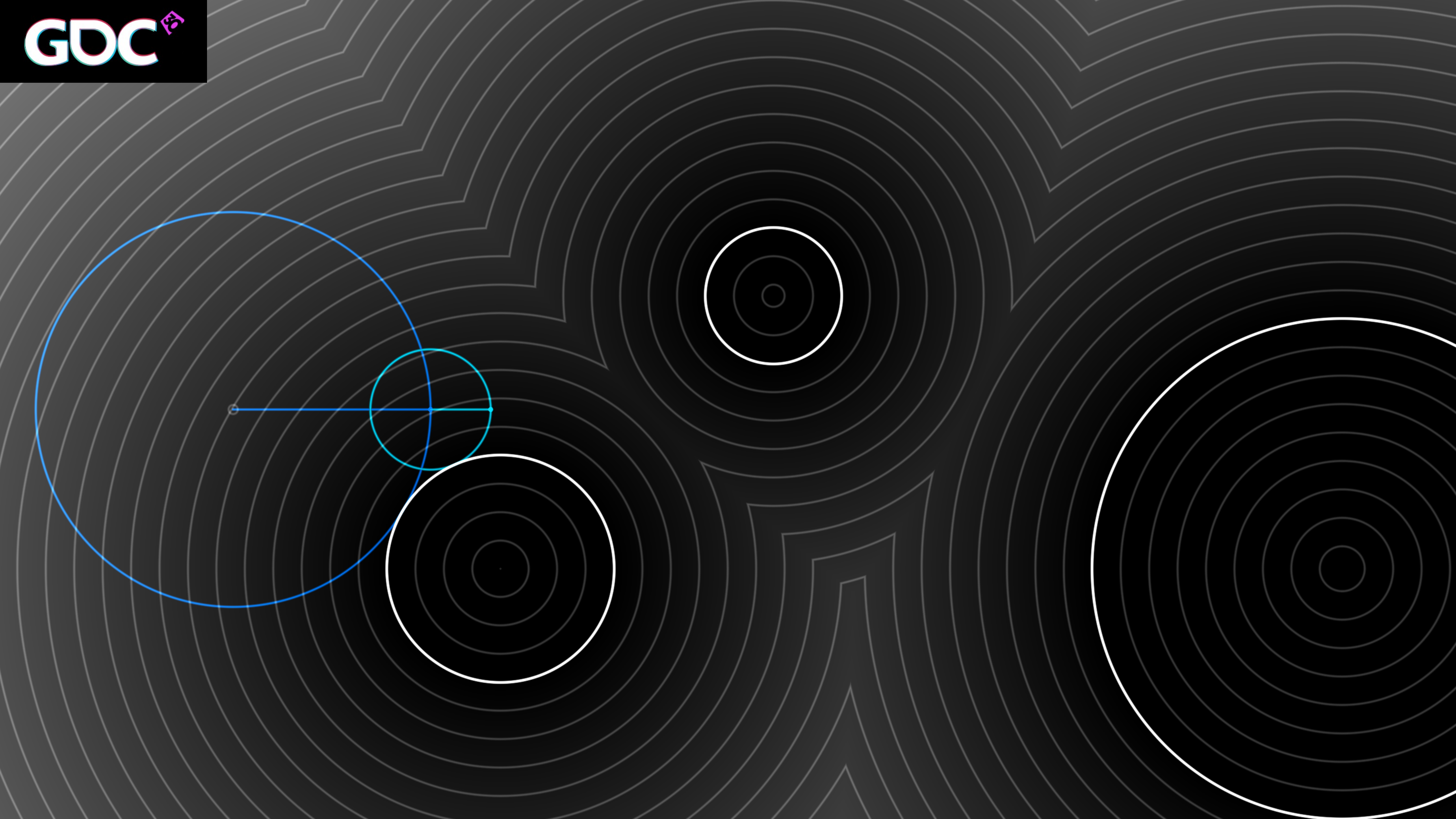




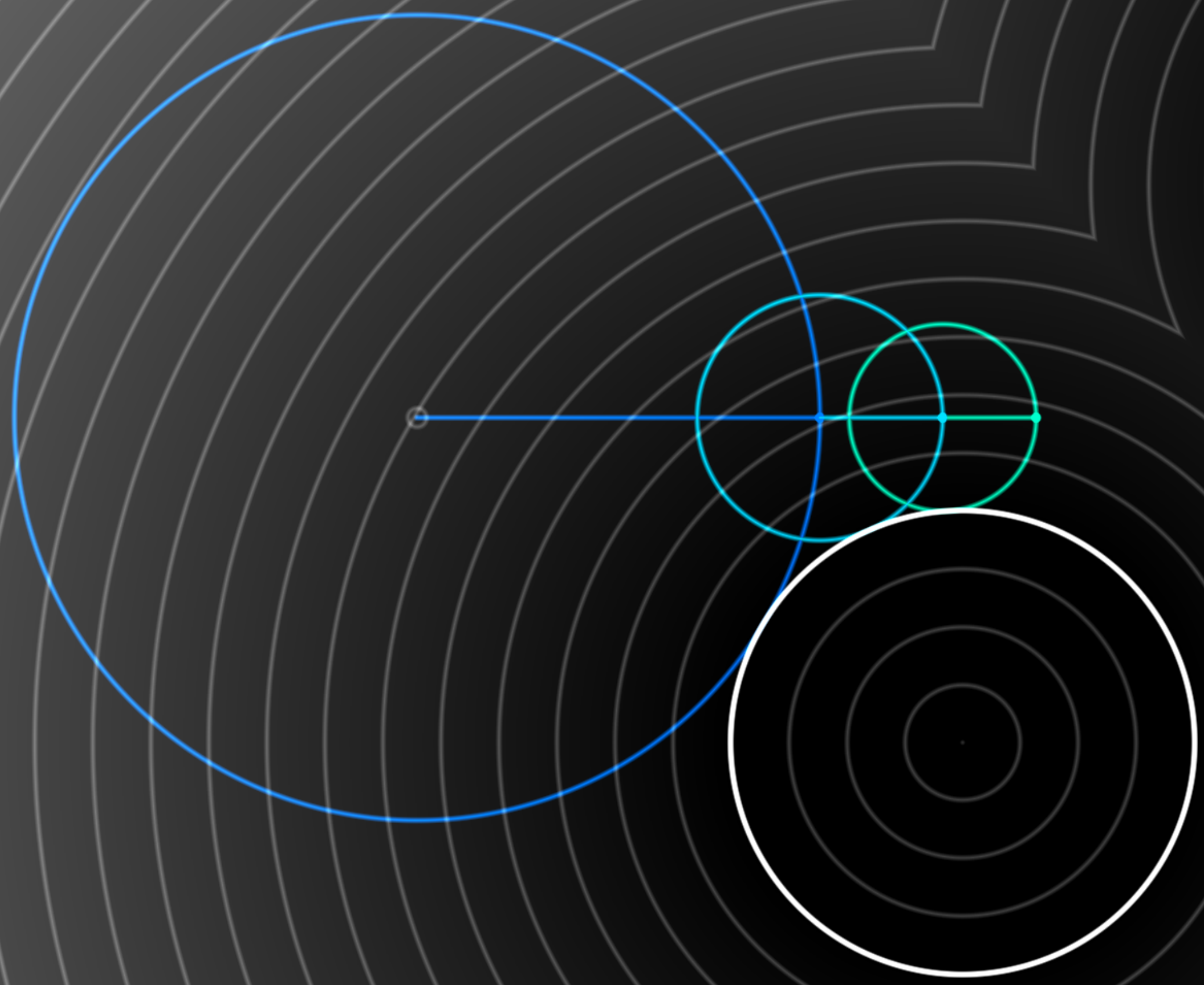




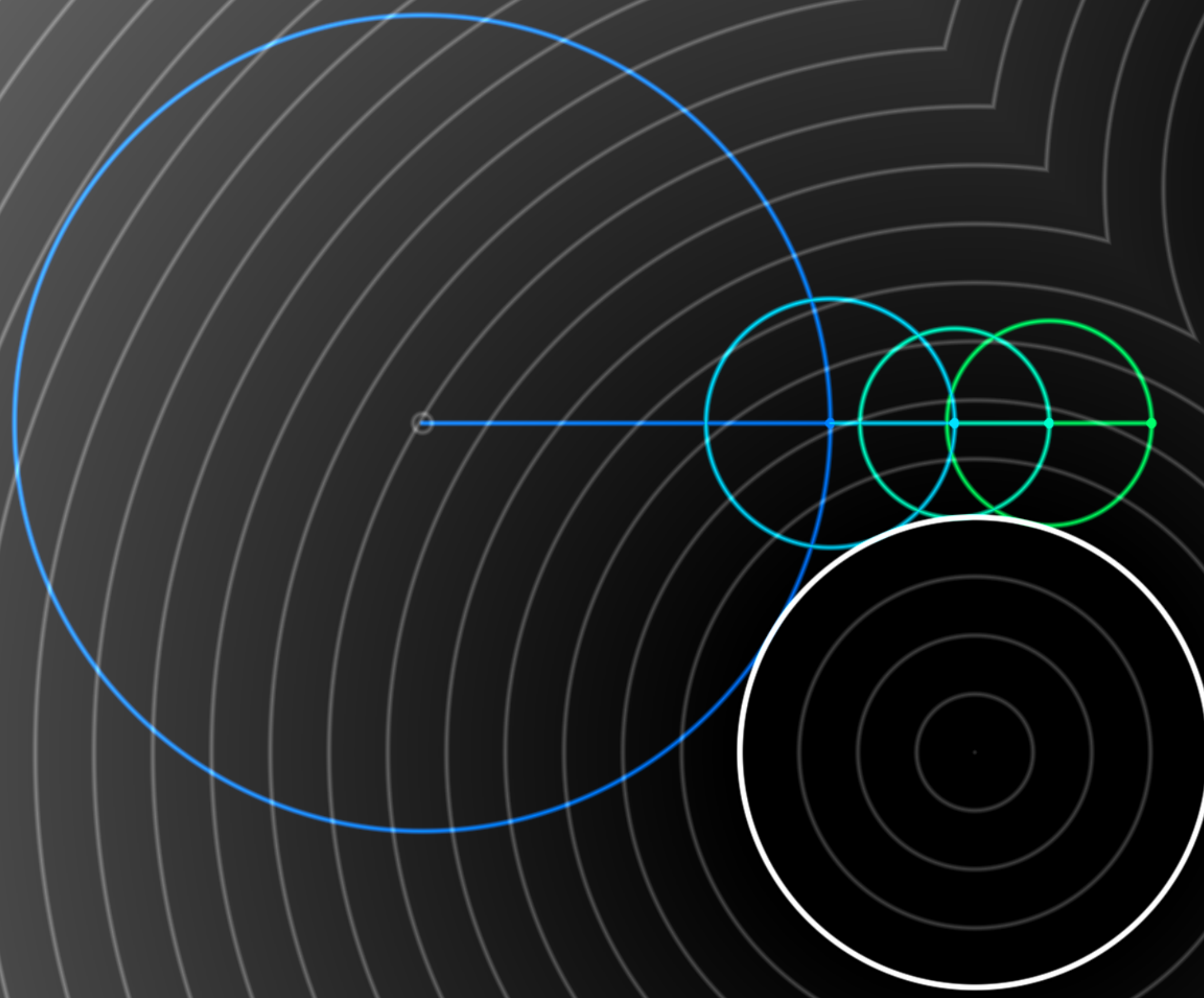




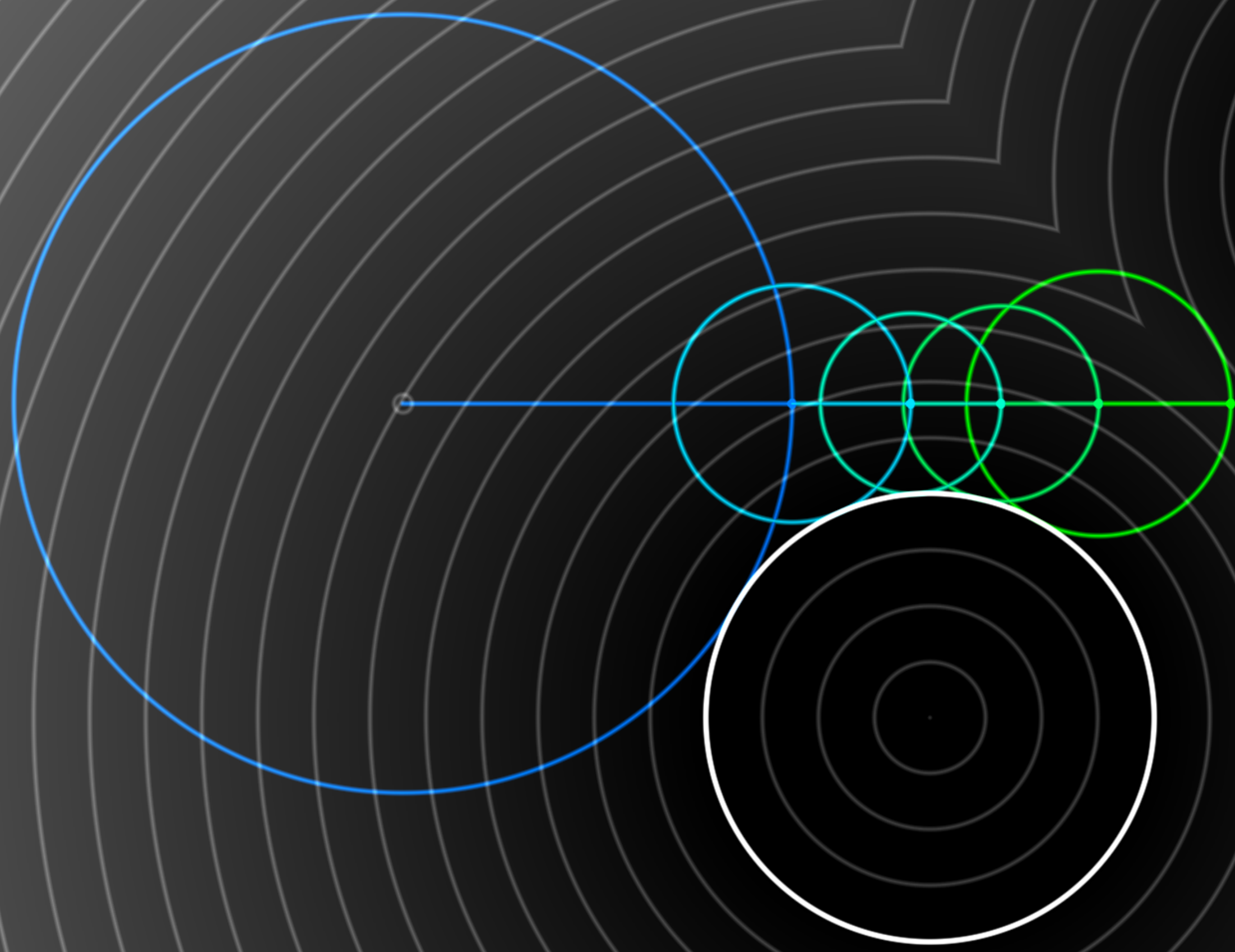




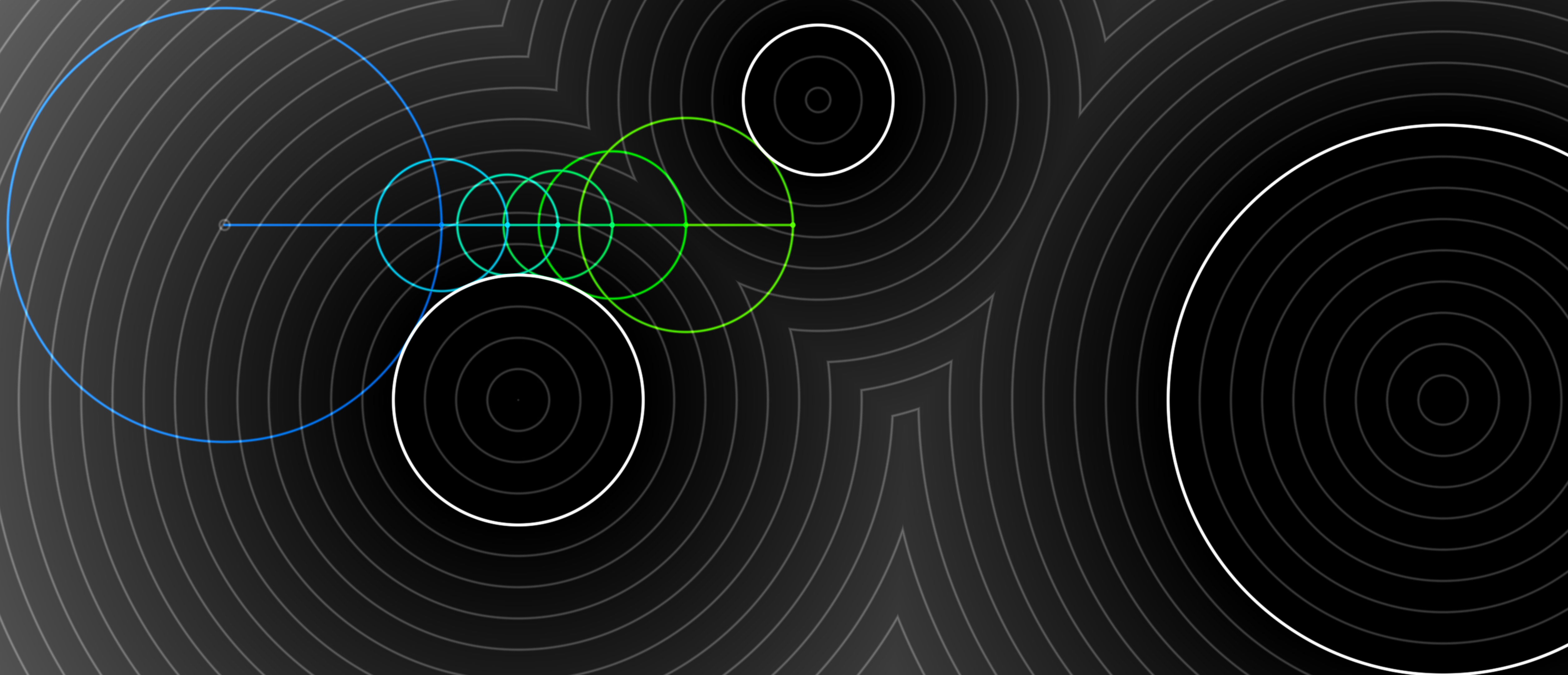




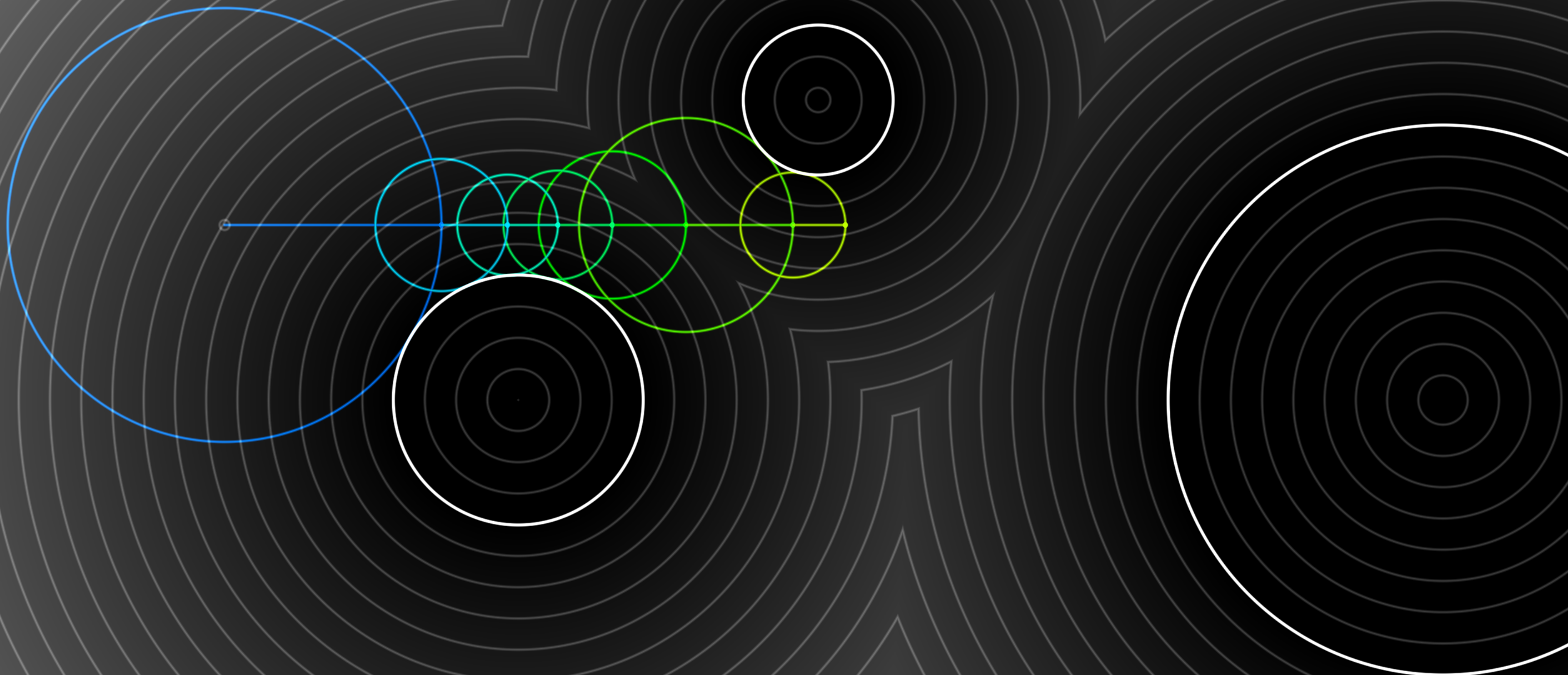




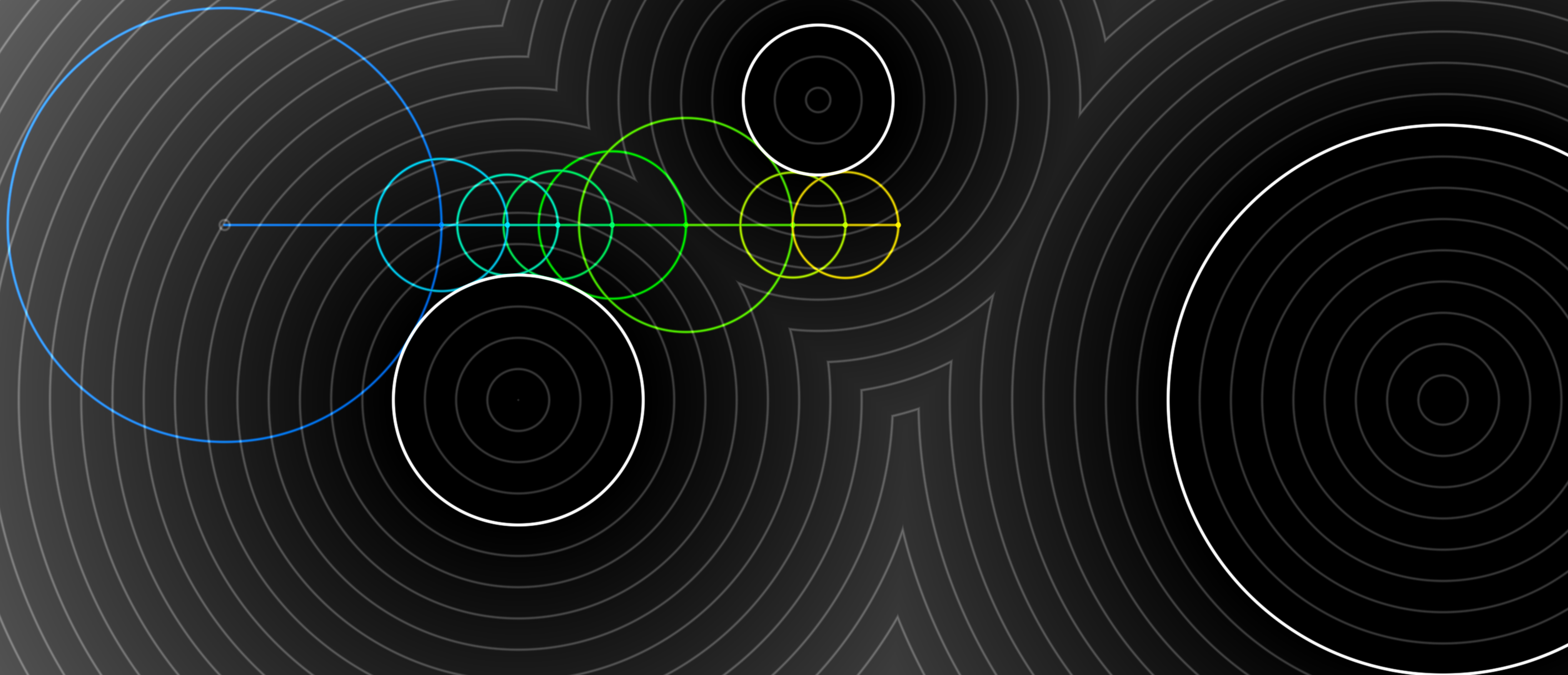




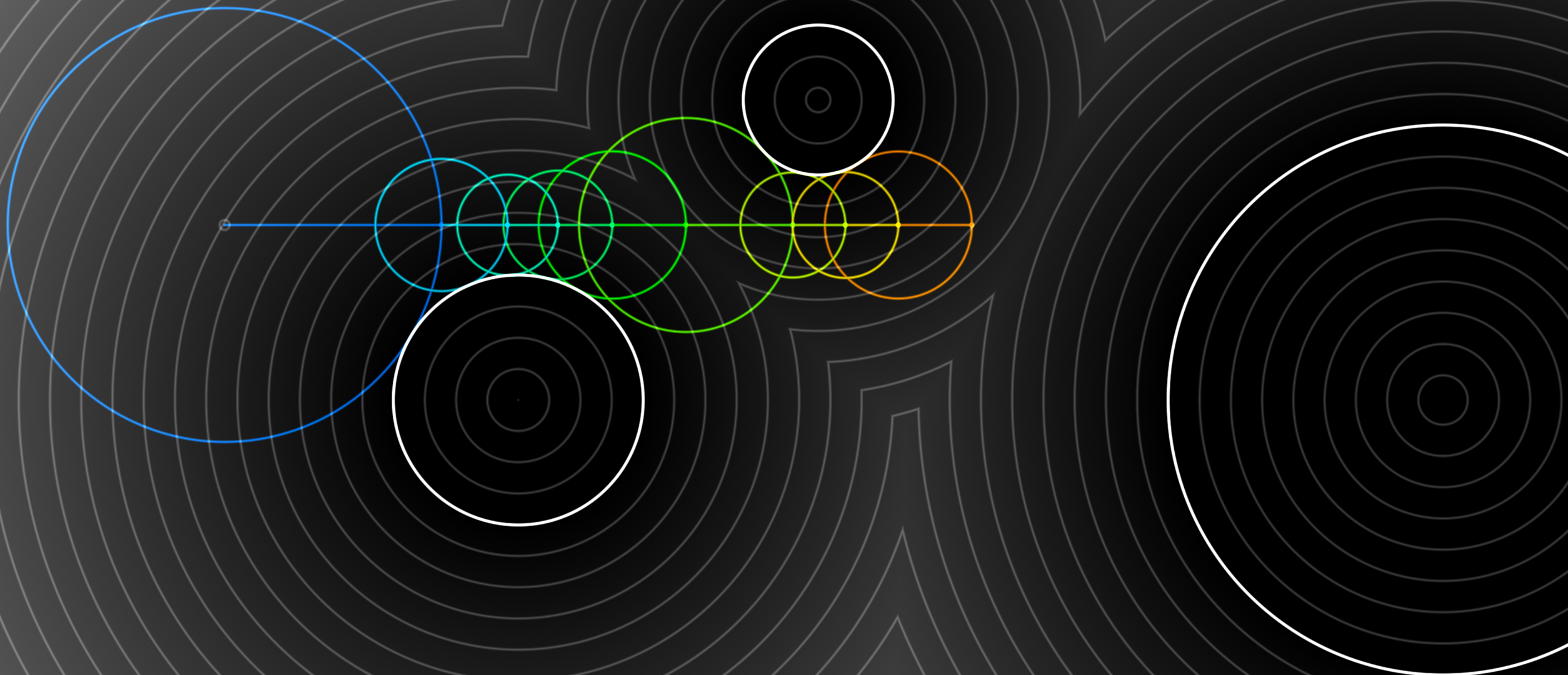




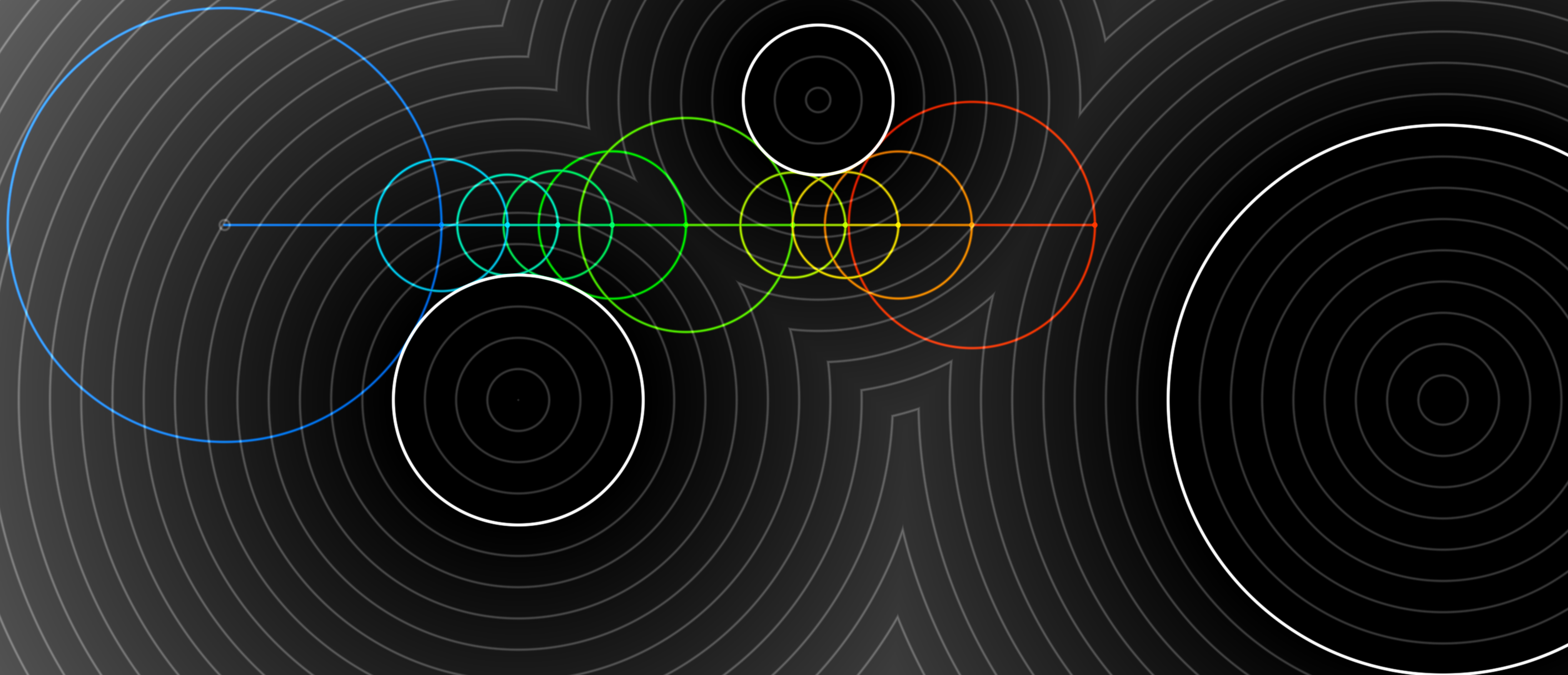




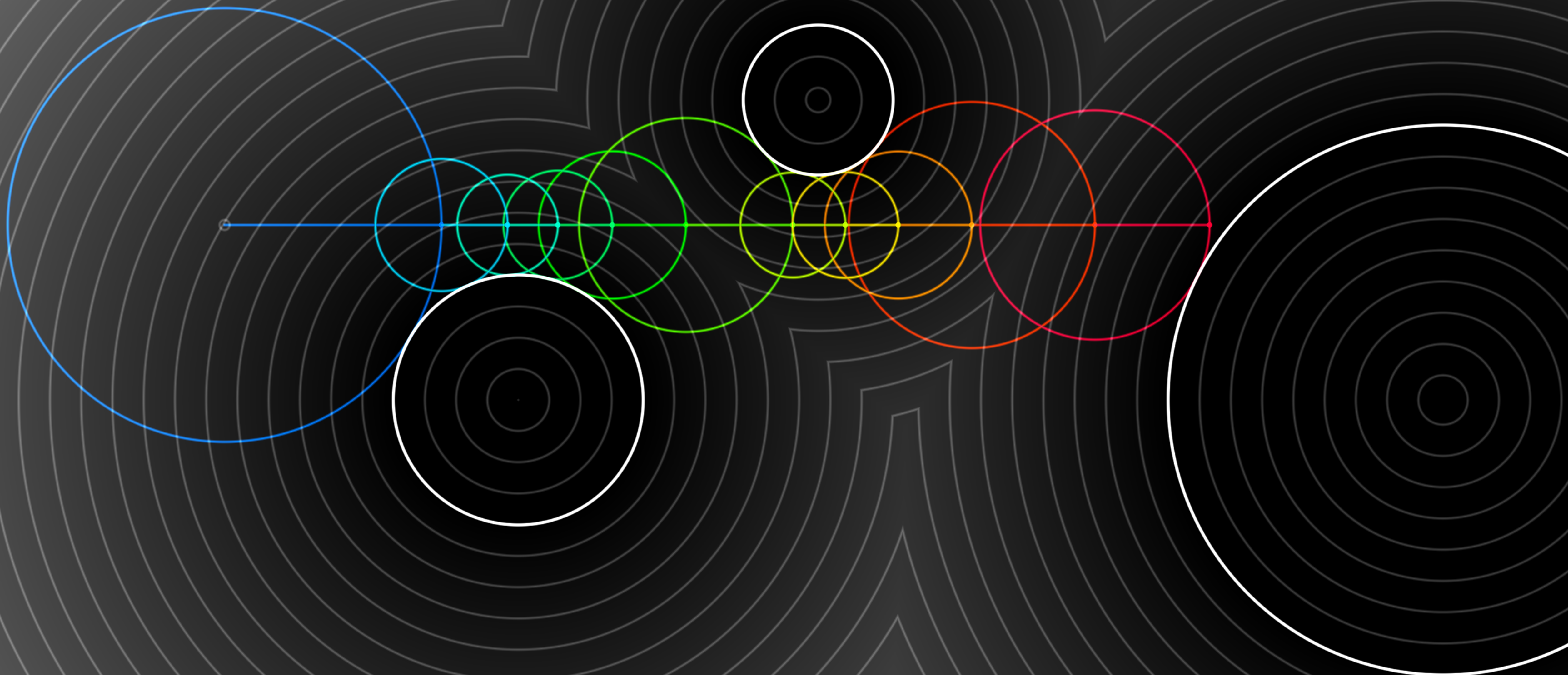




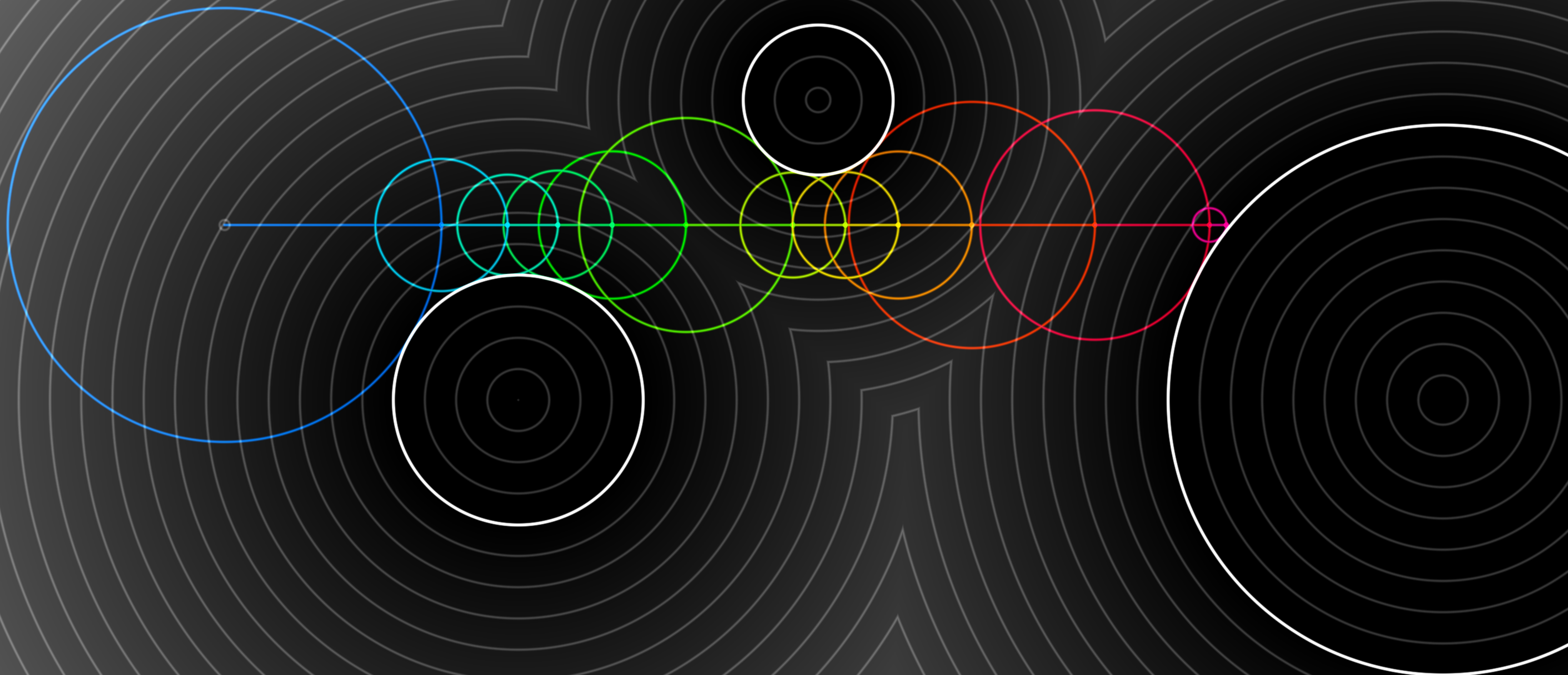




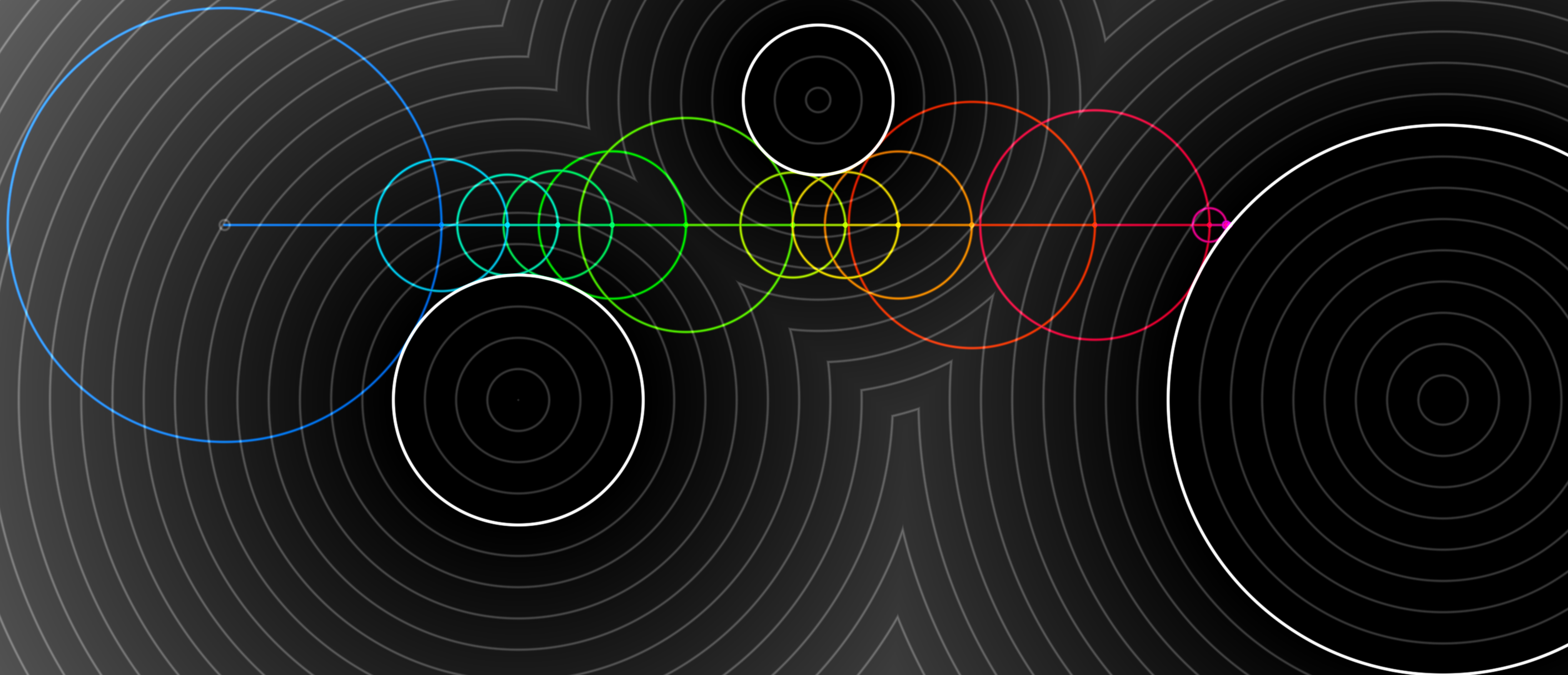




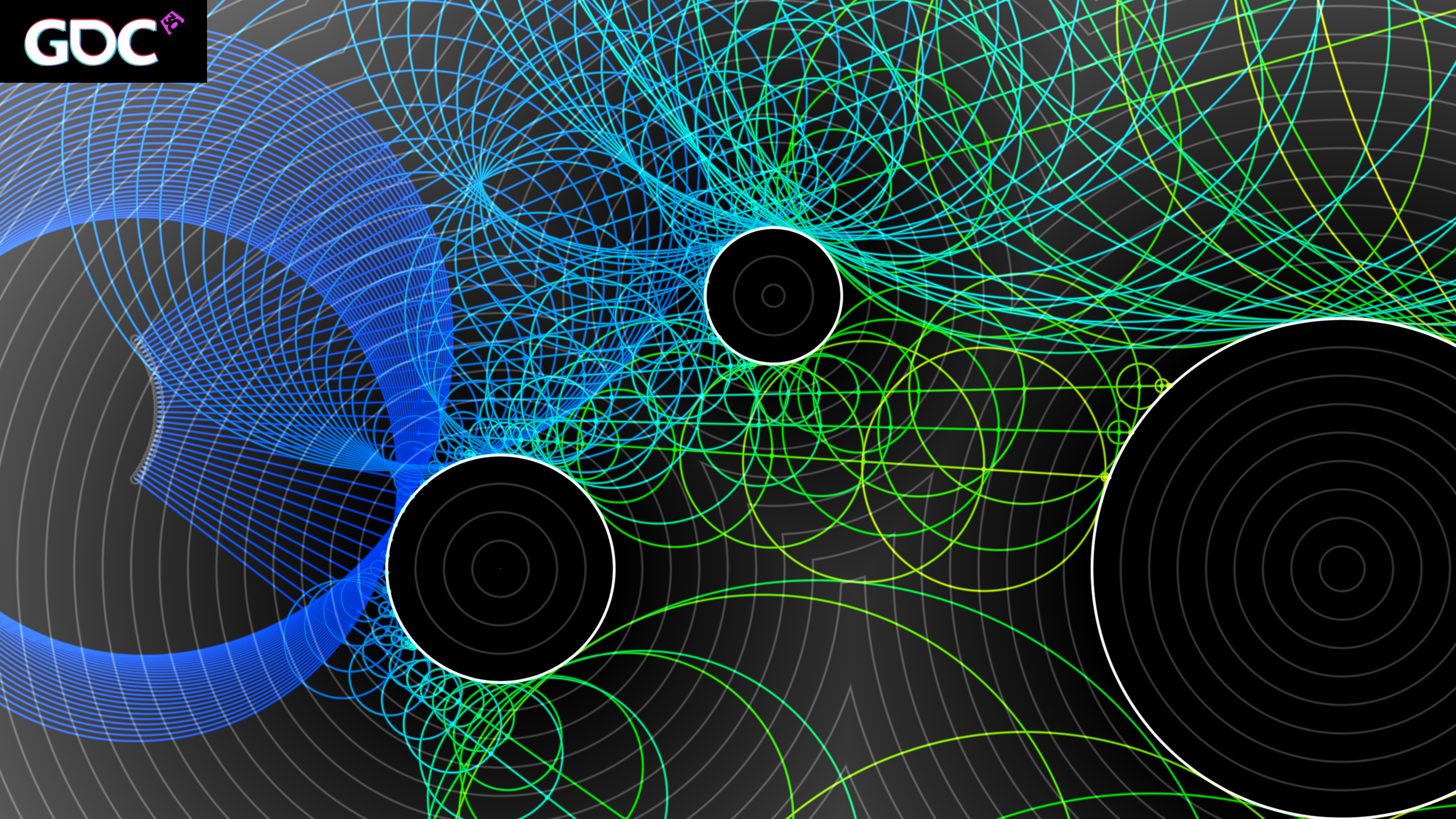




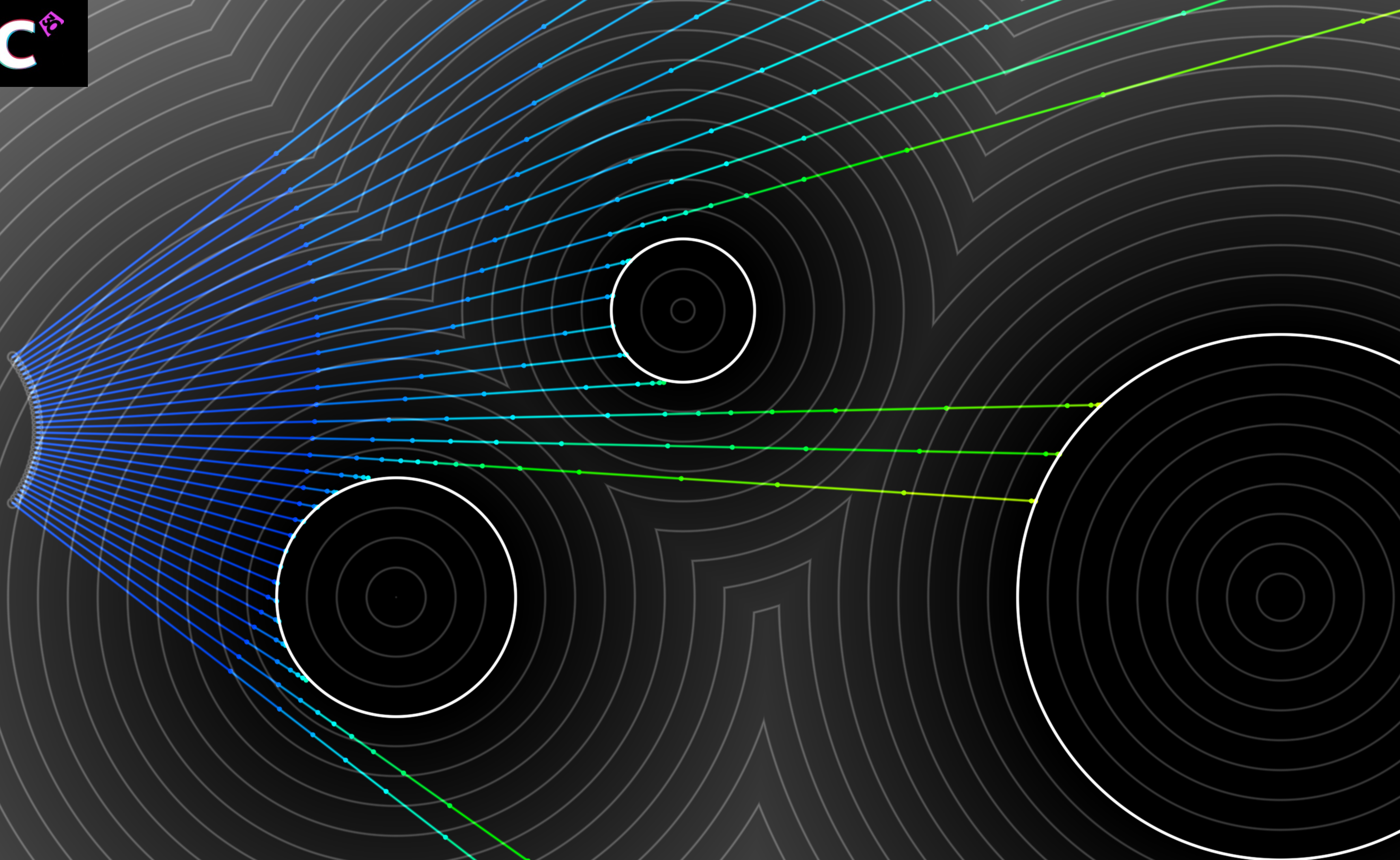




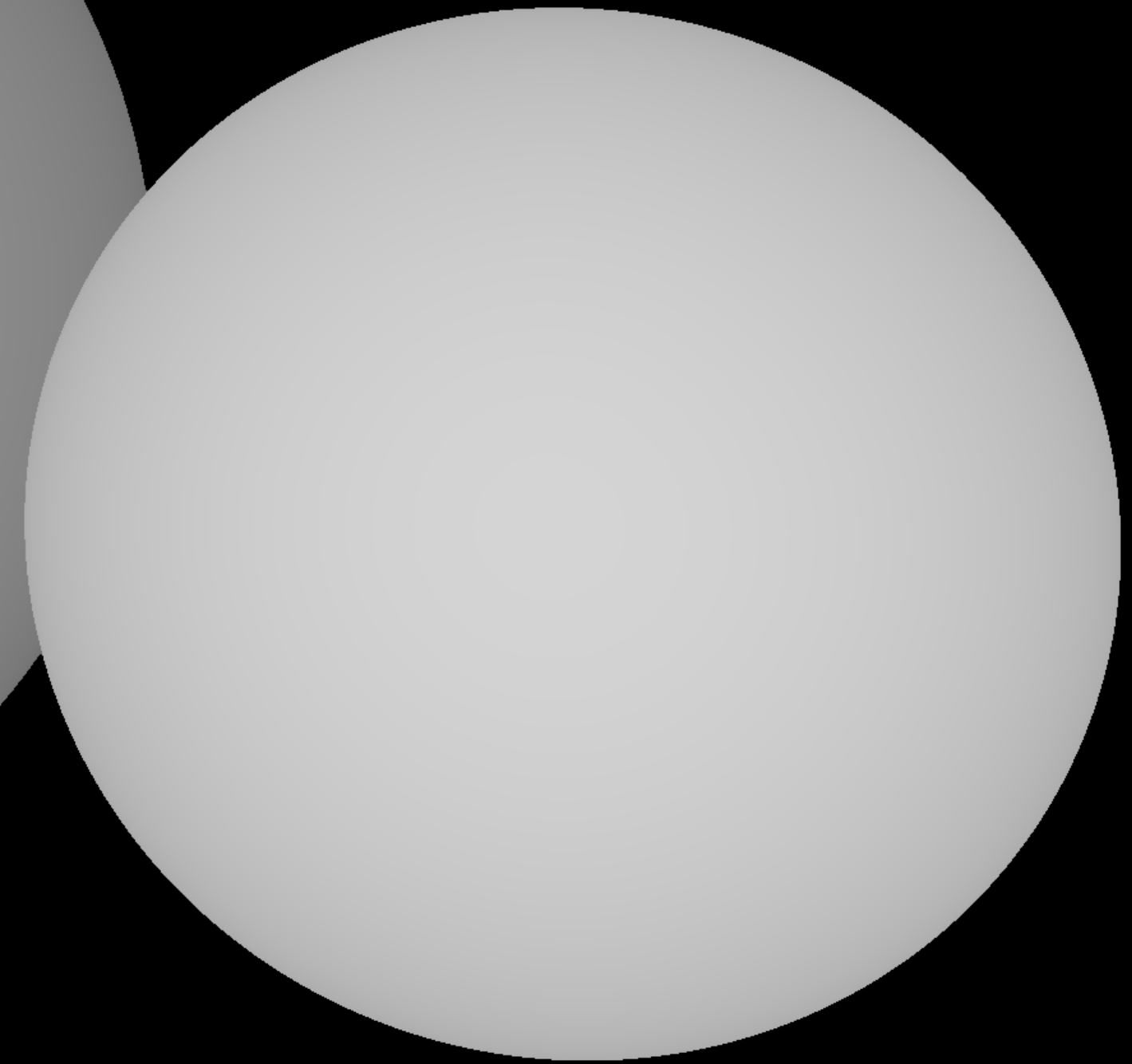
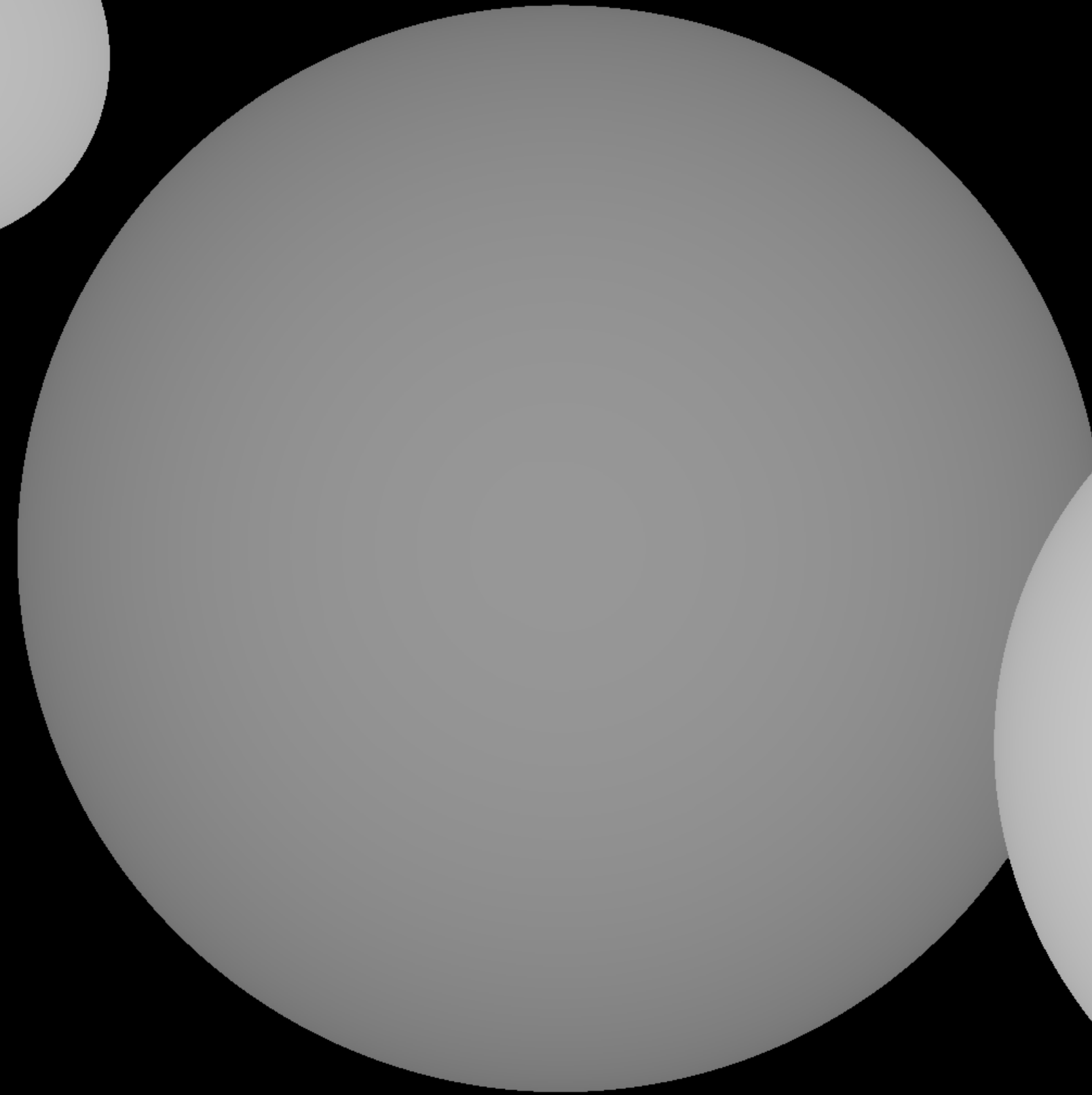
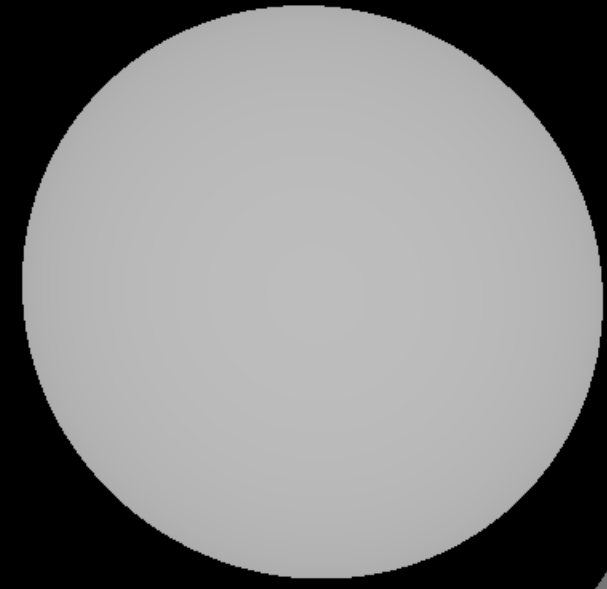








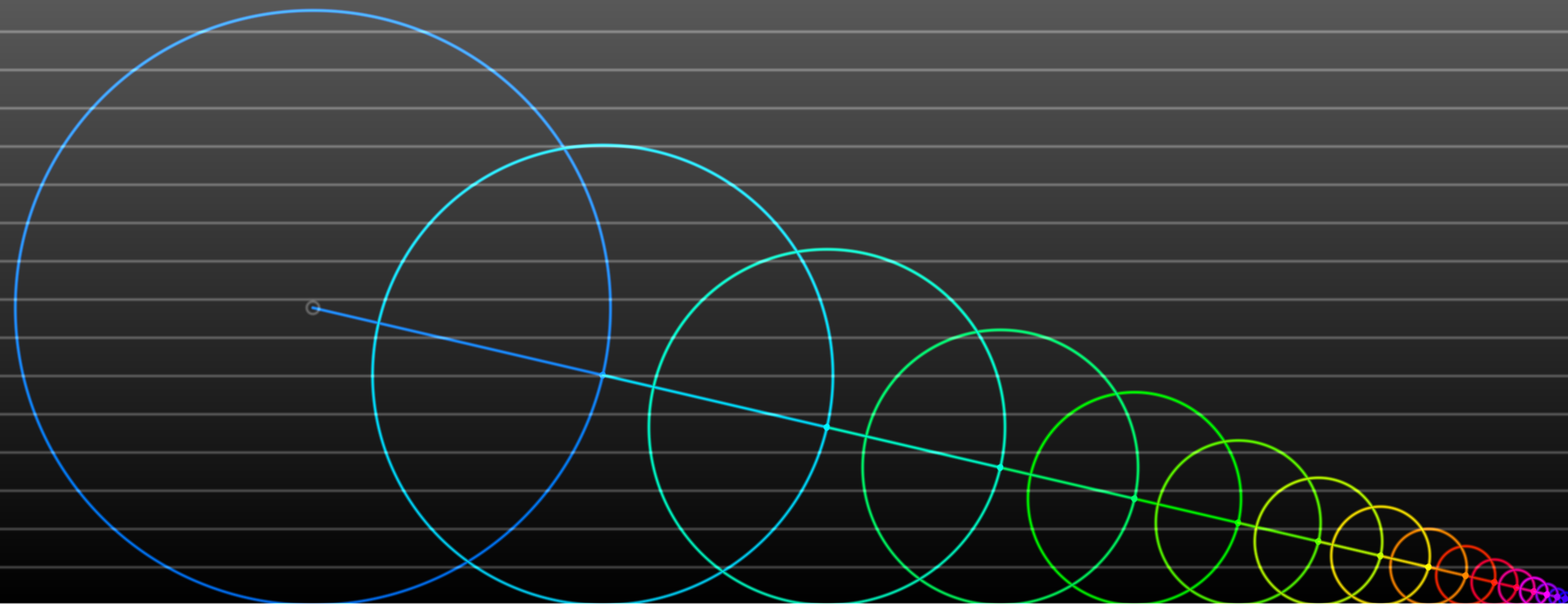




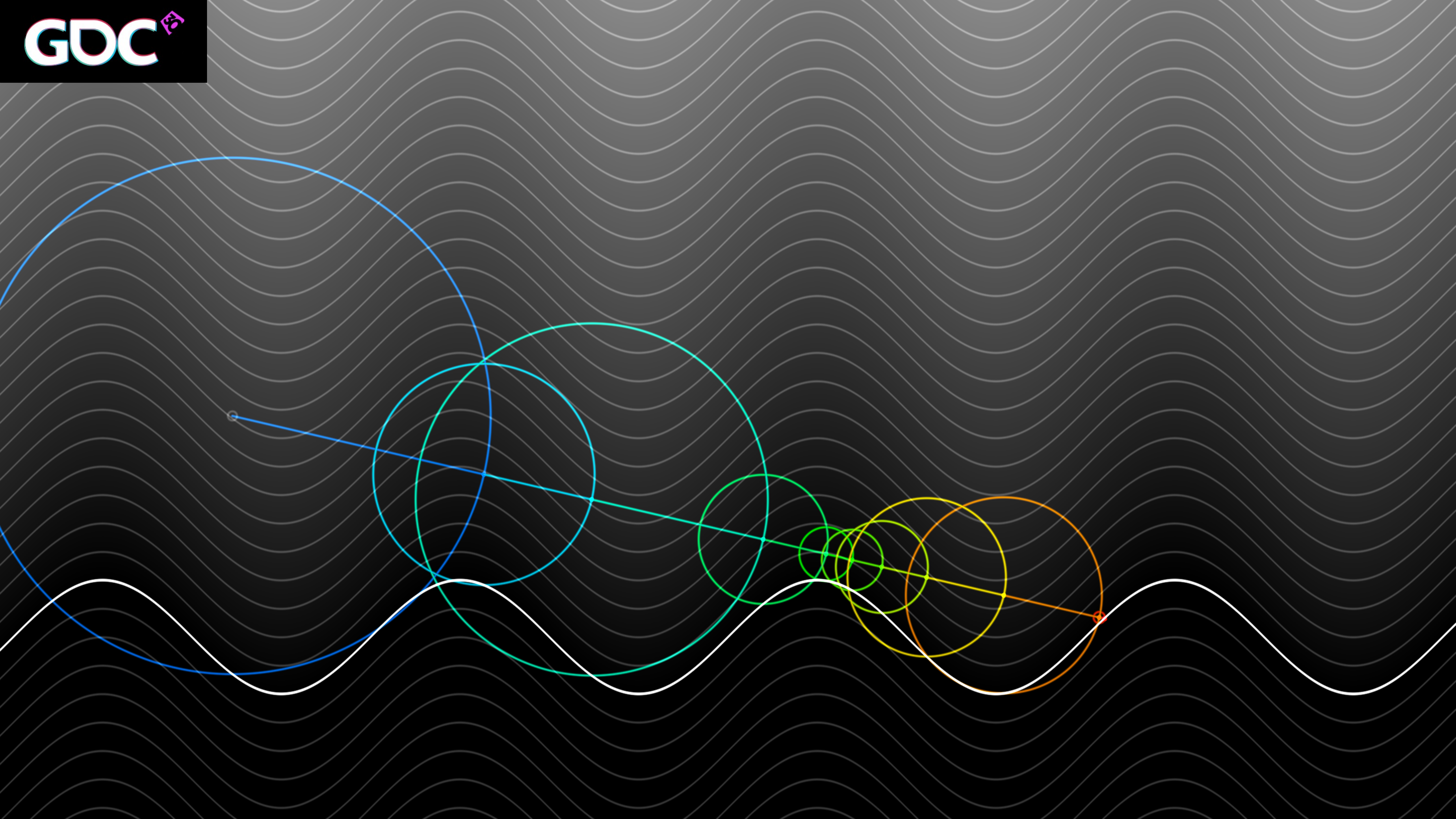




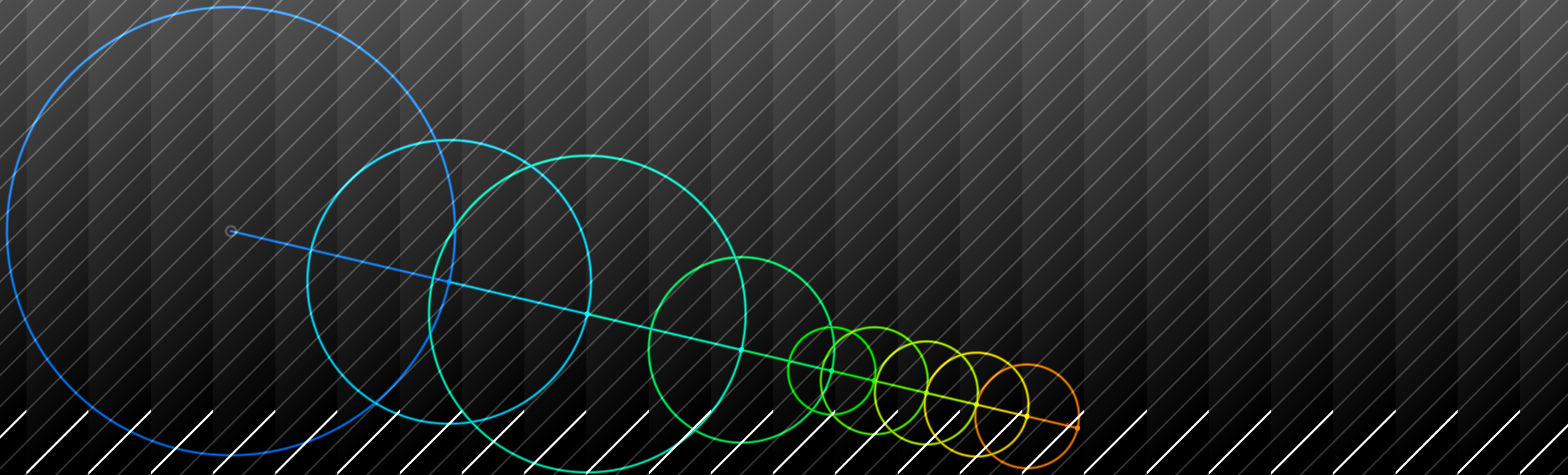




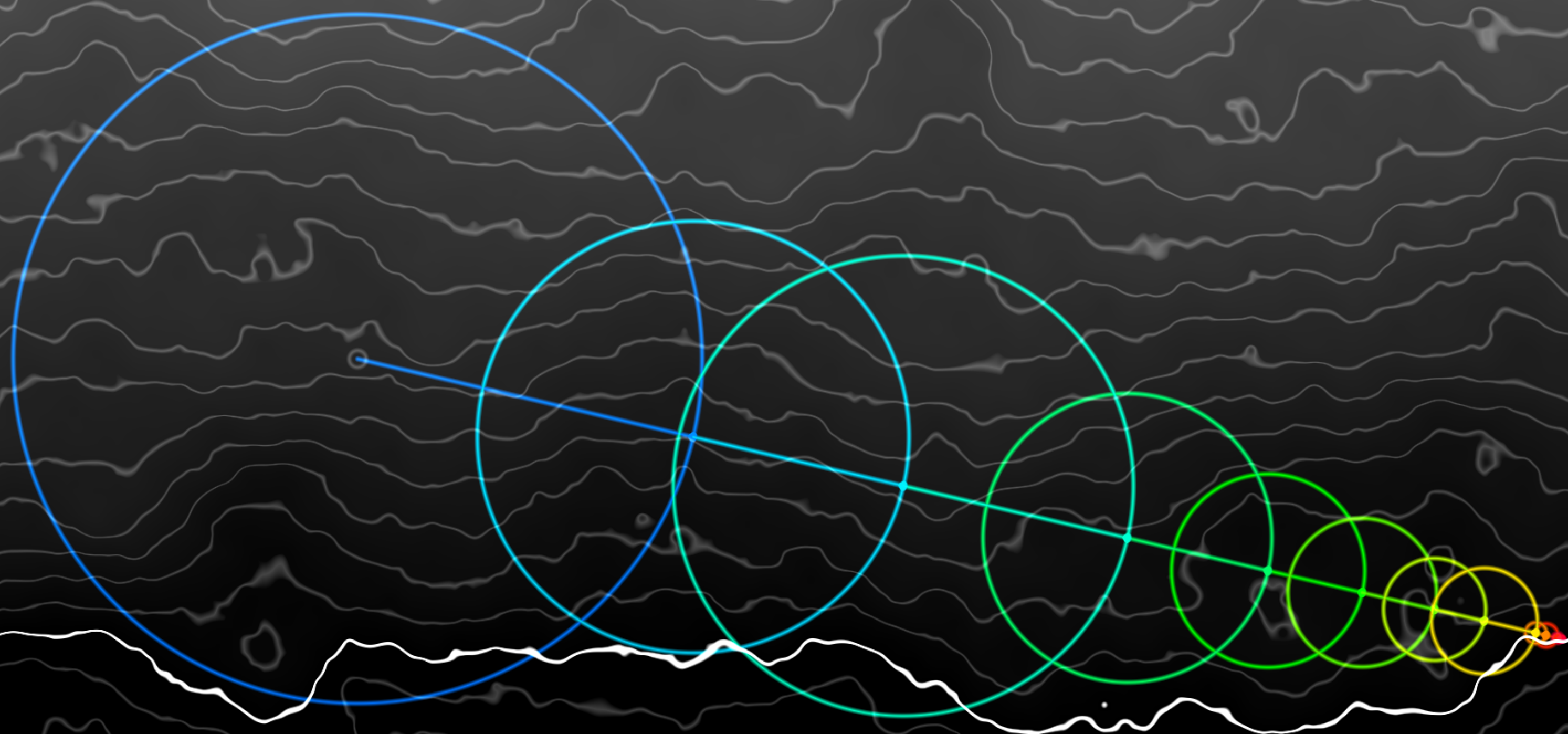




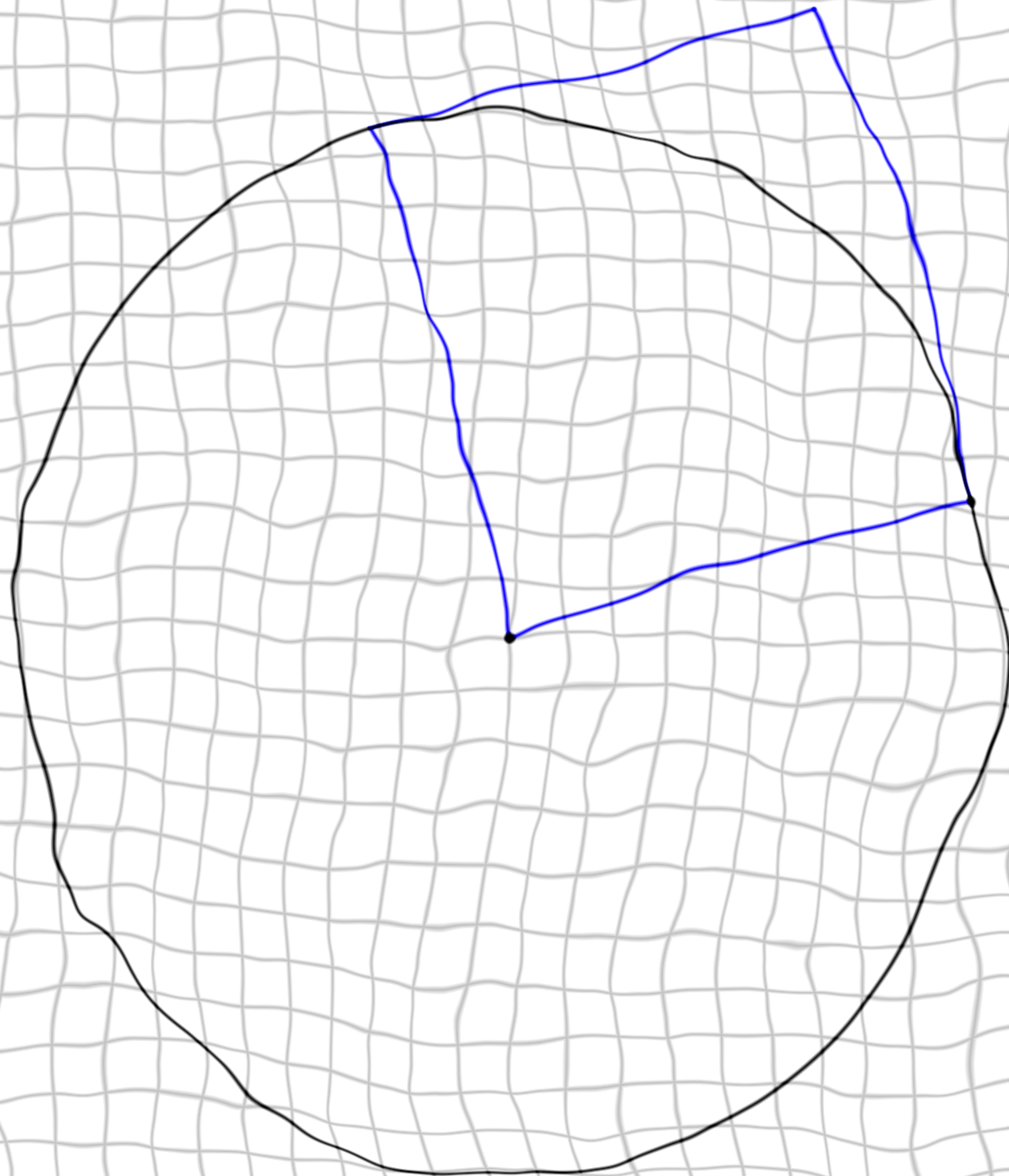




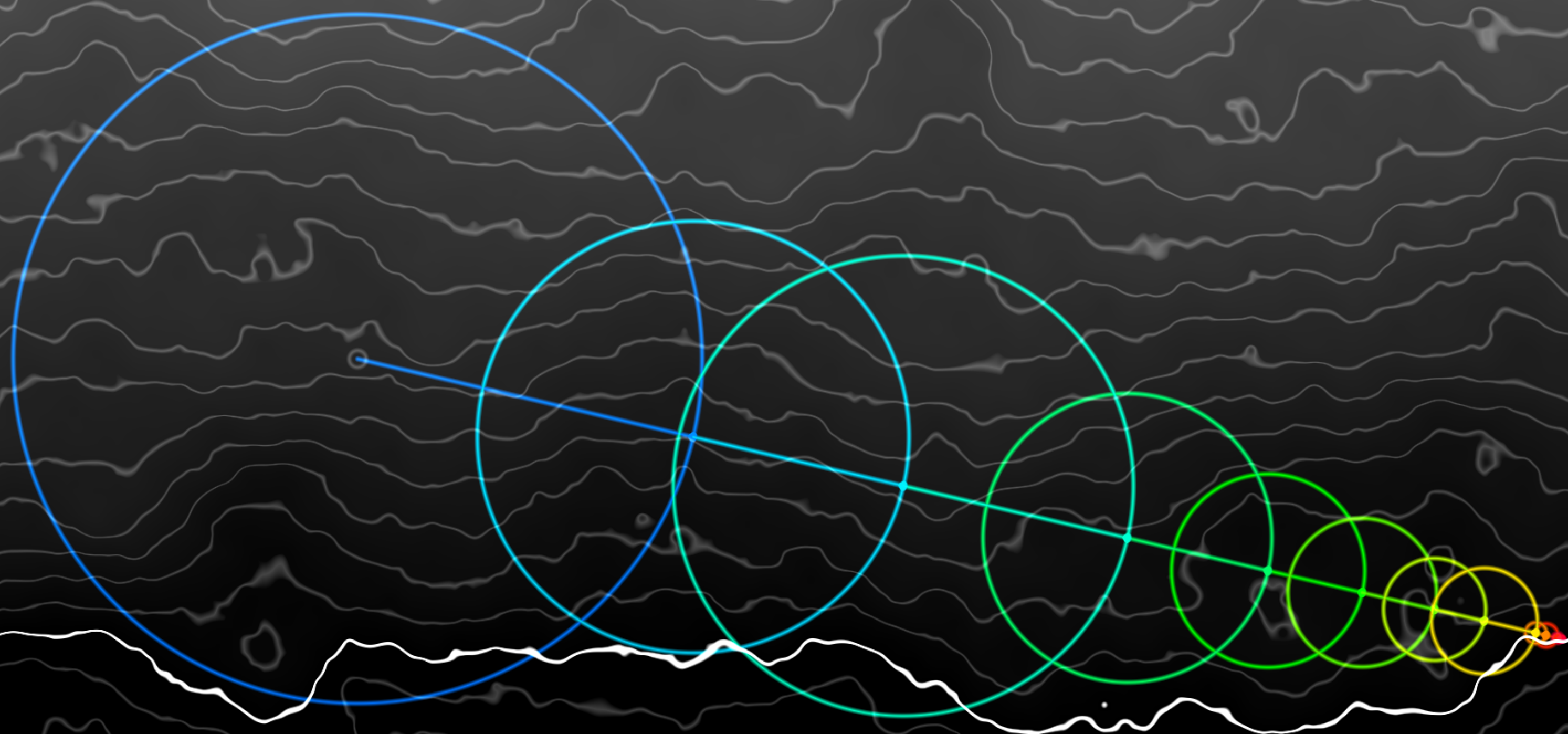








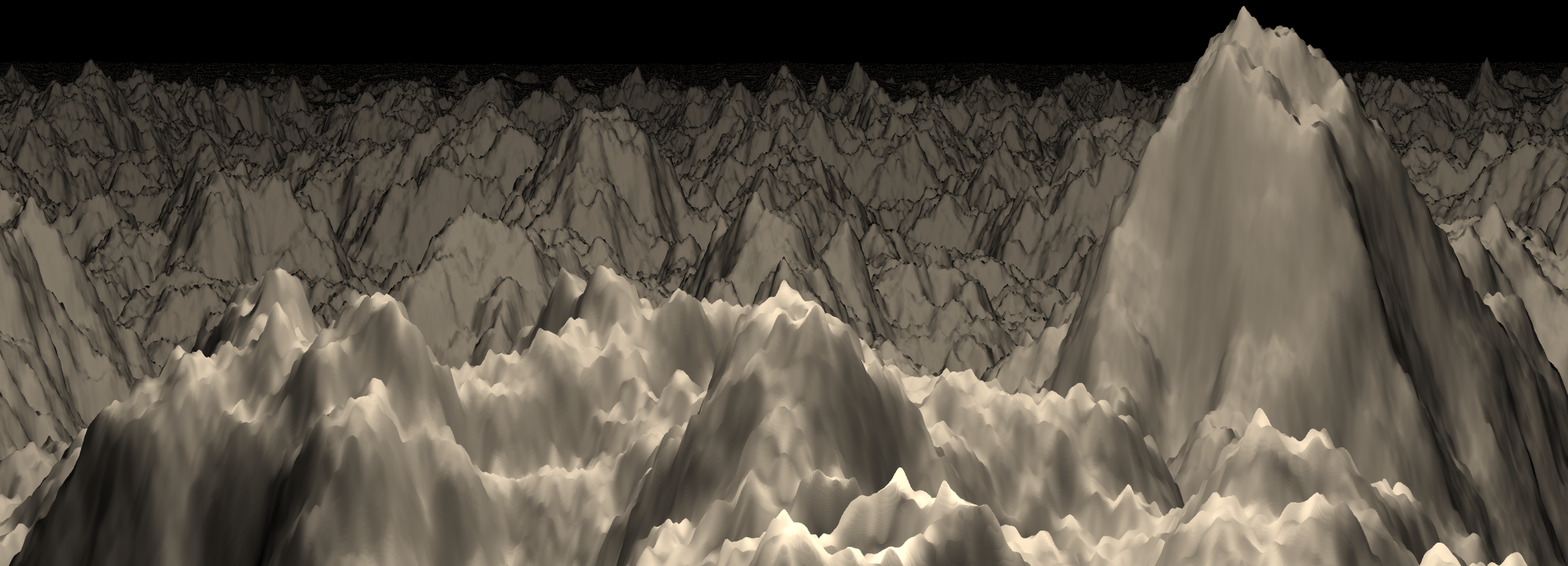




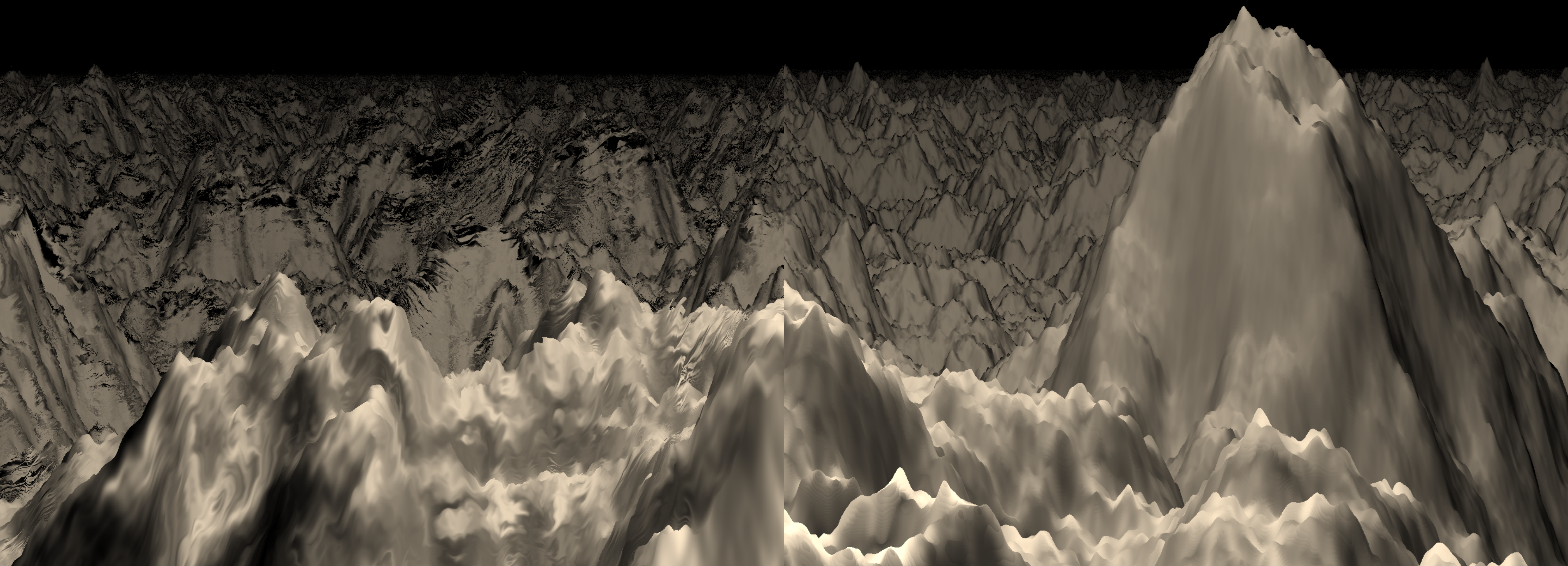


Imagine this is the space we will  
render into.

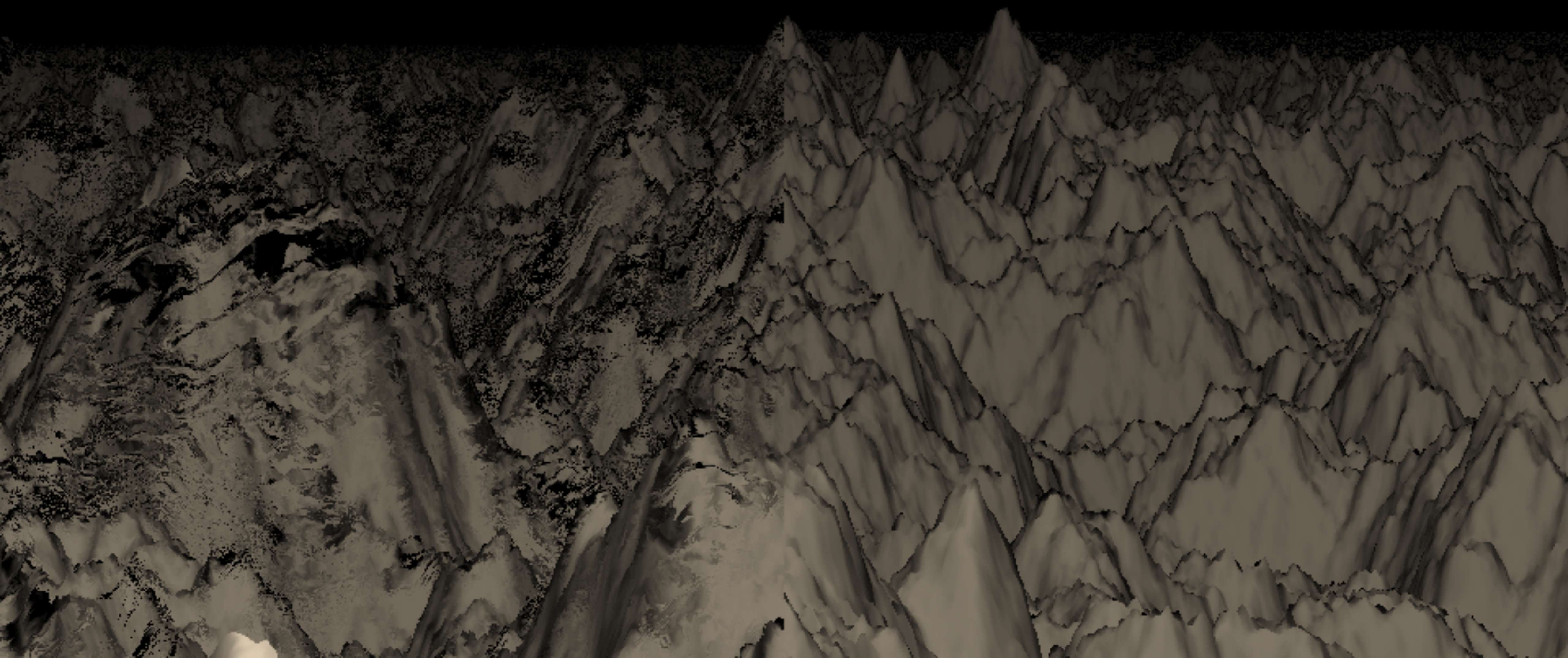










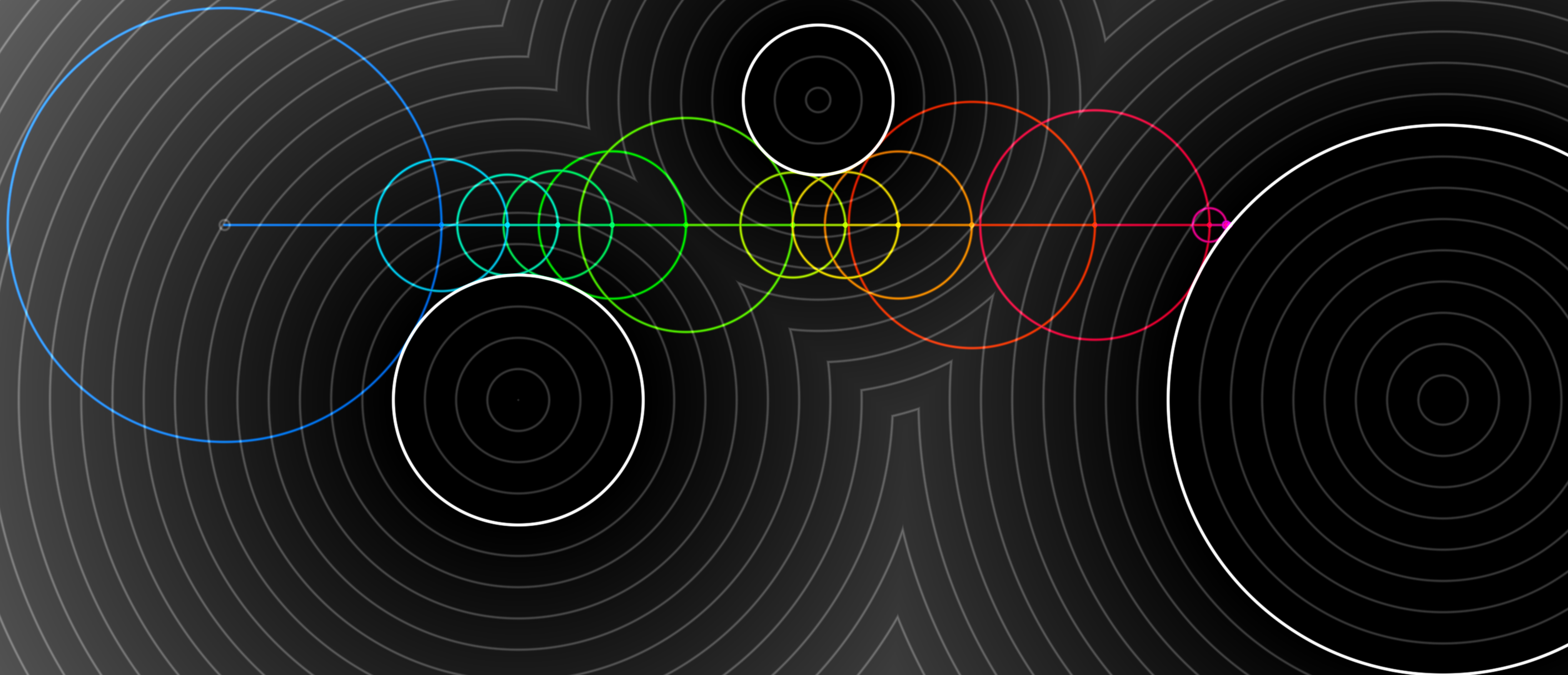




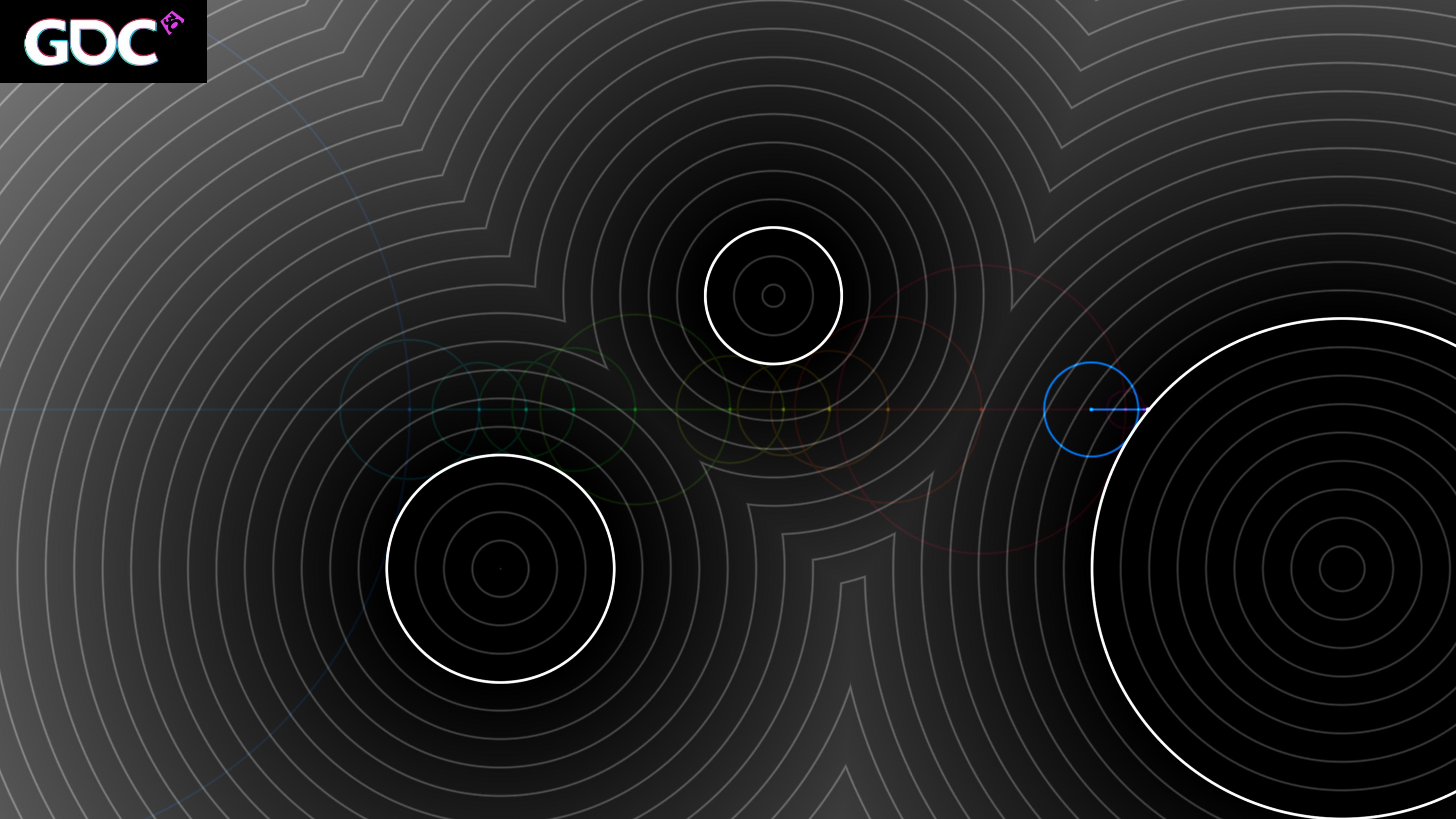
# Normal



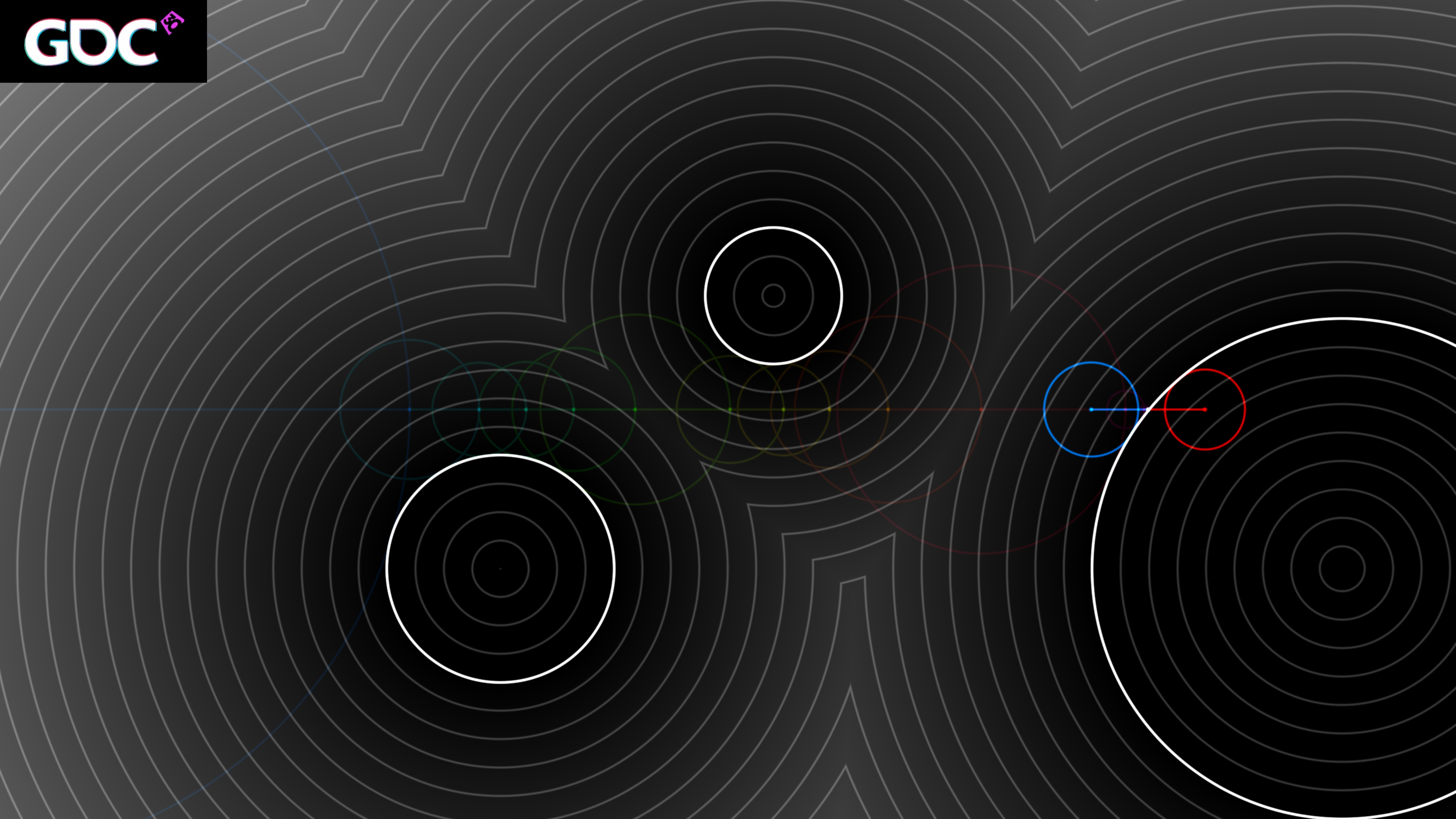




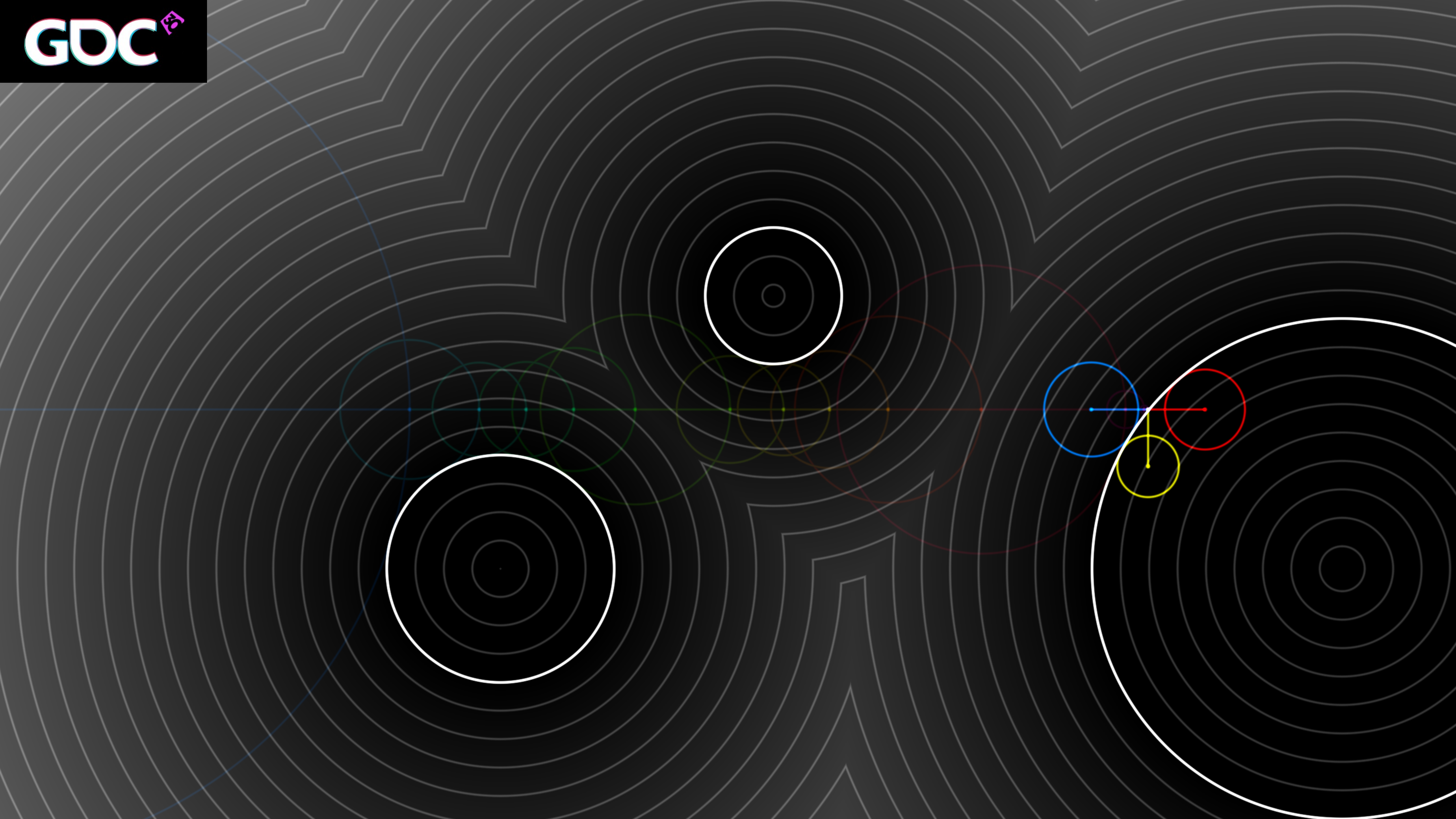




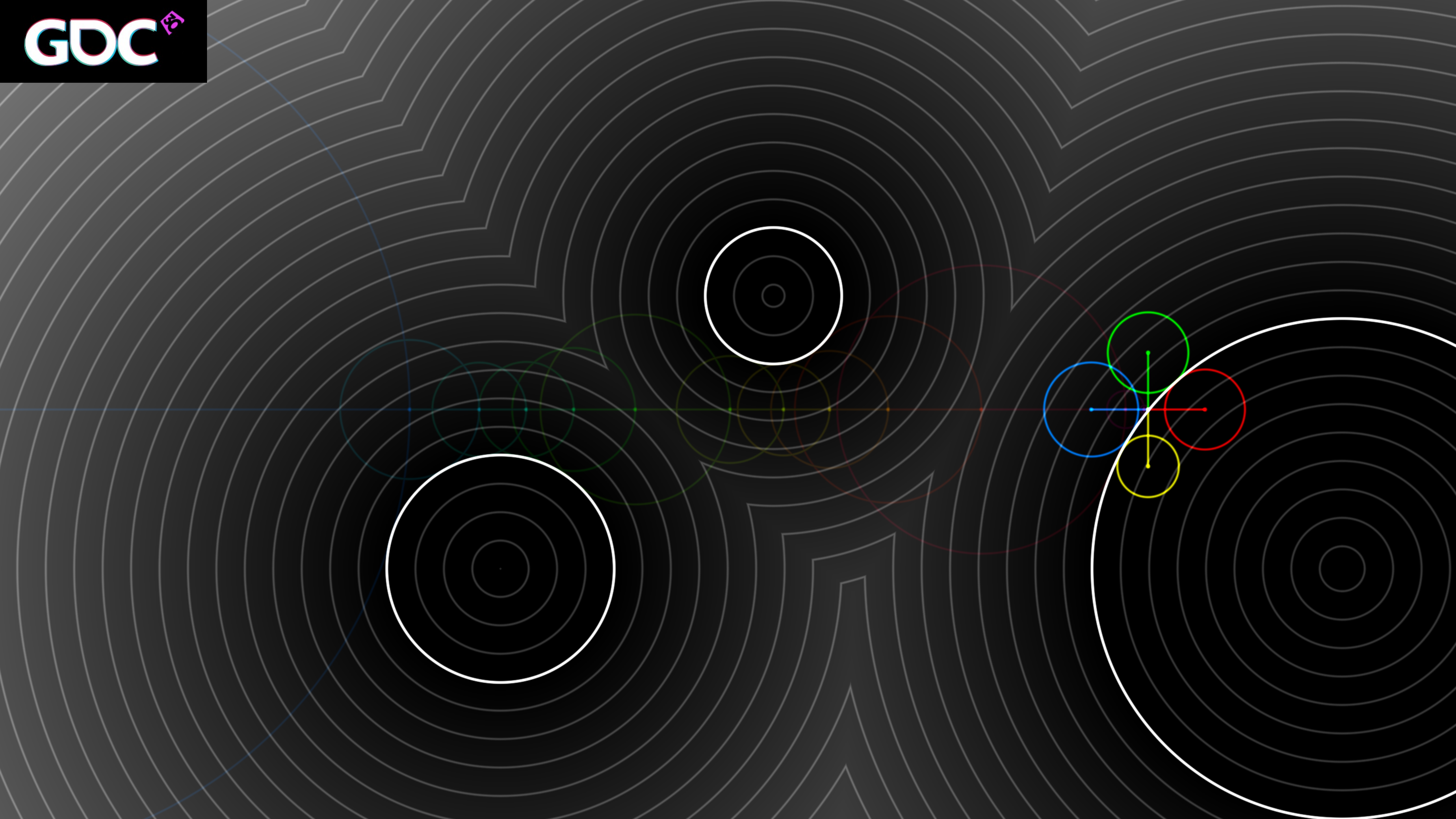




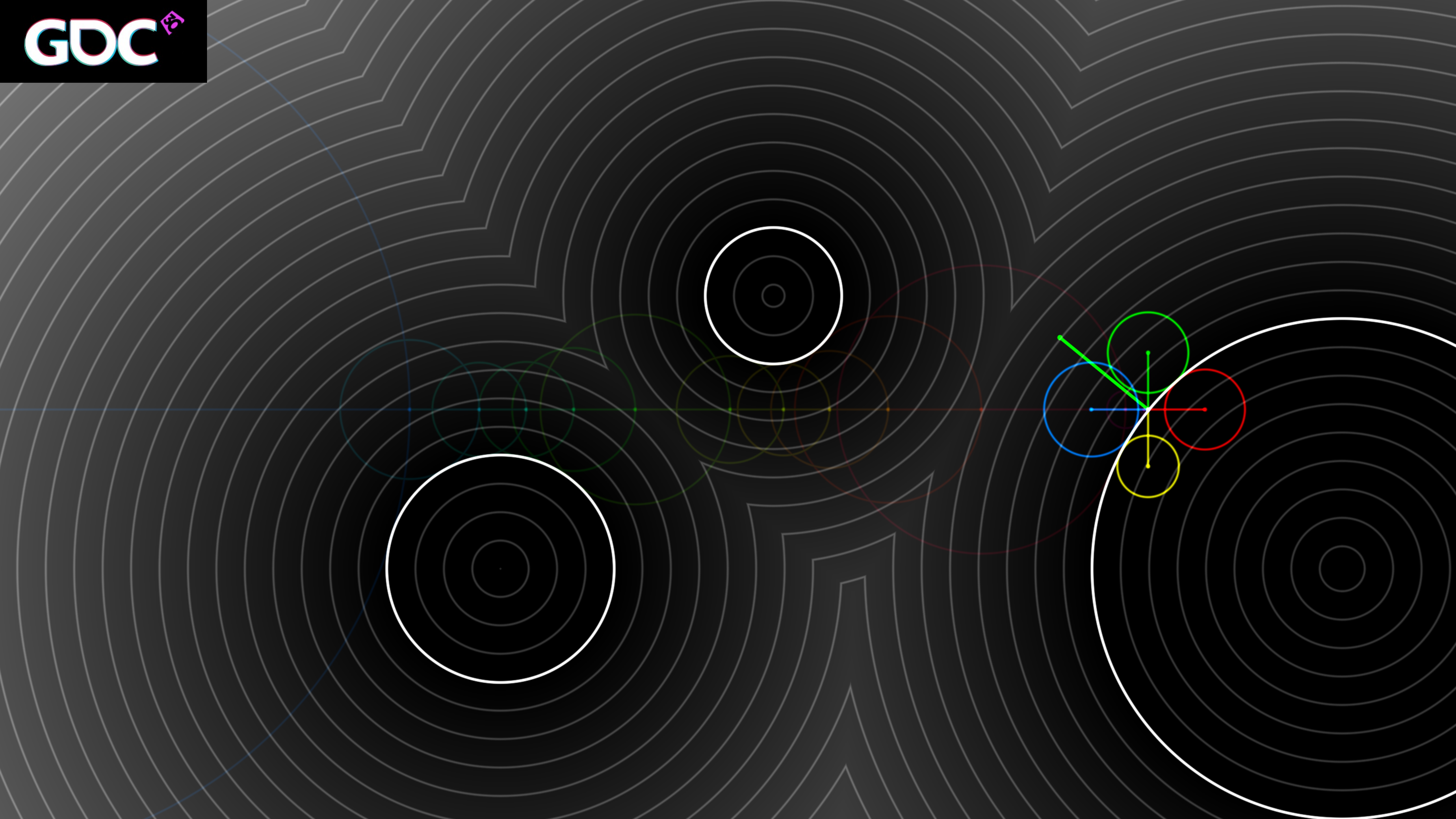




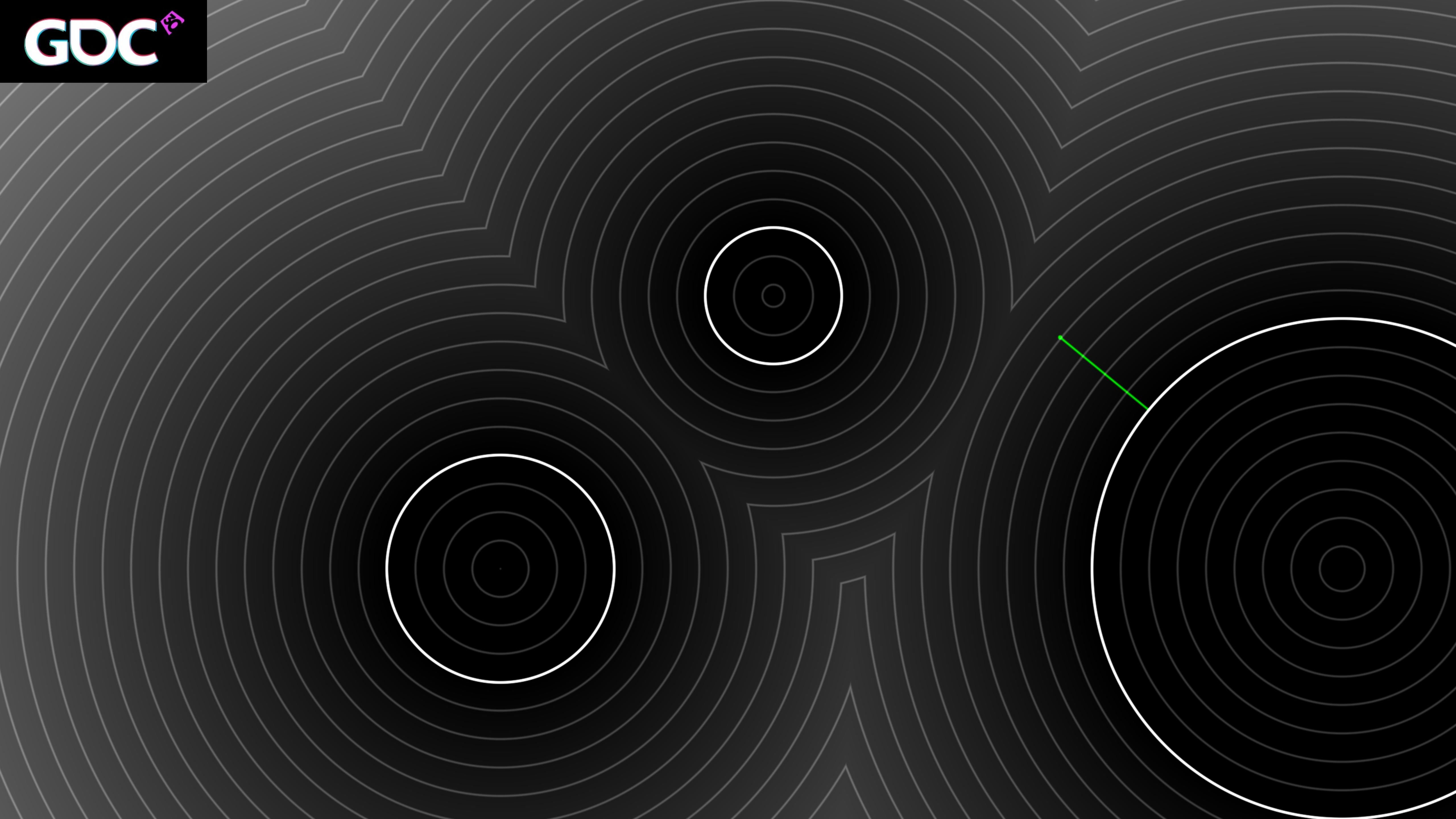




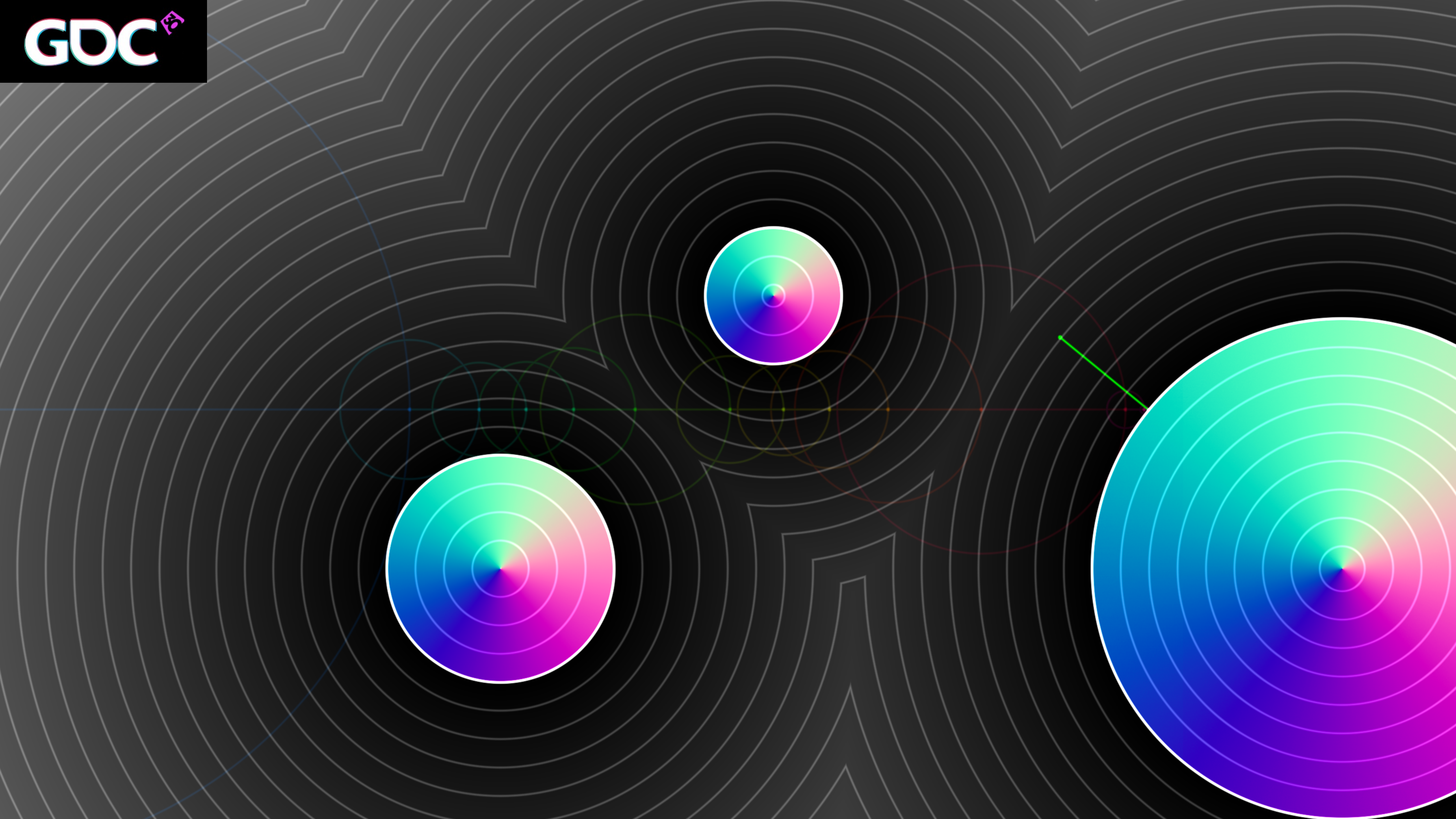




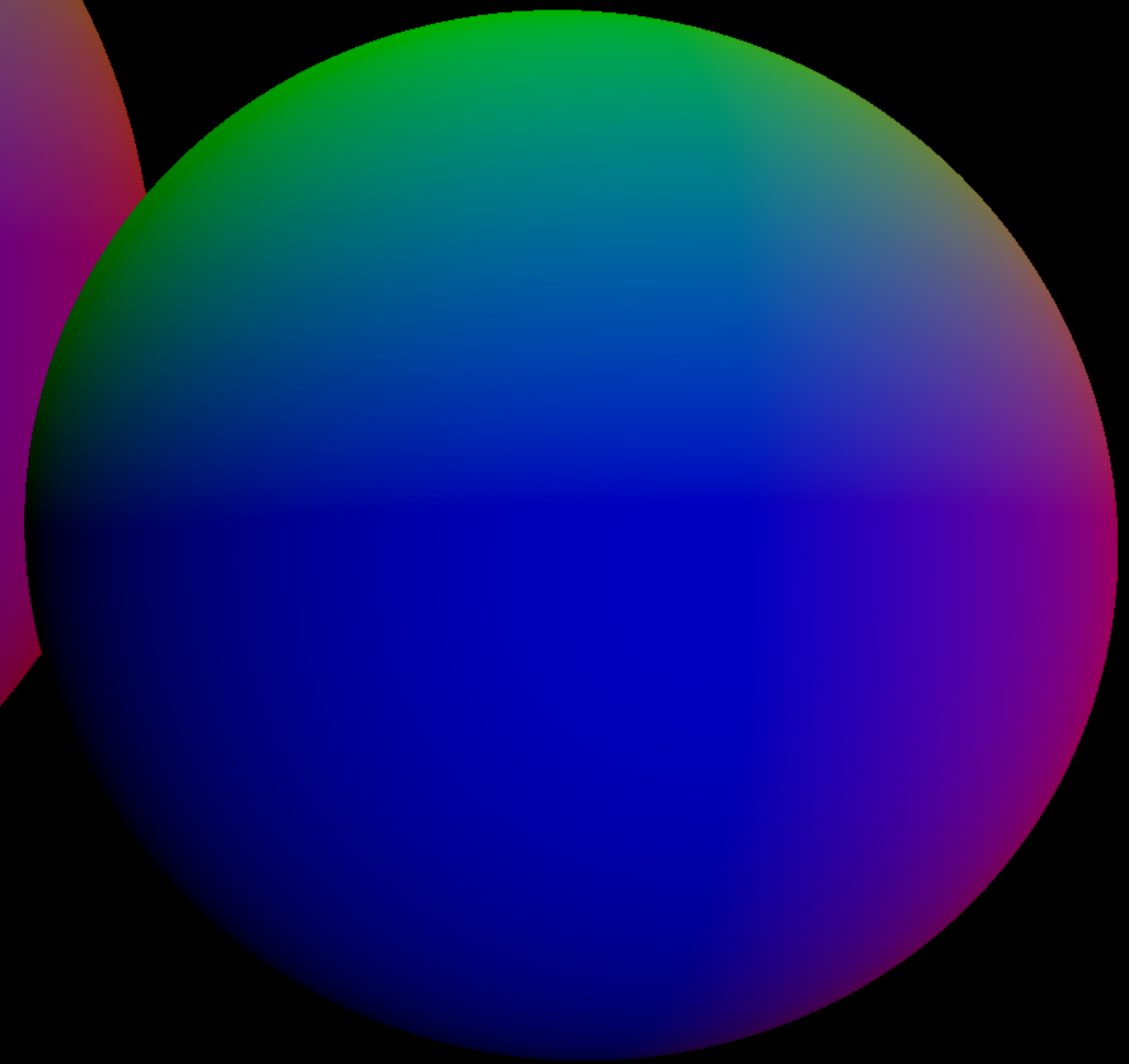
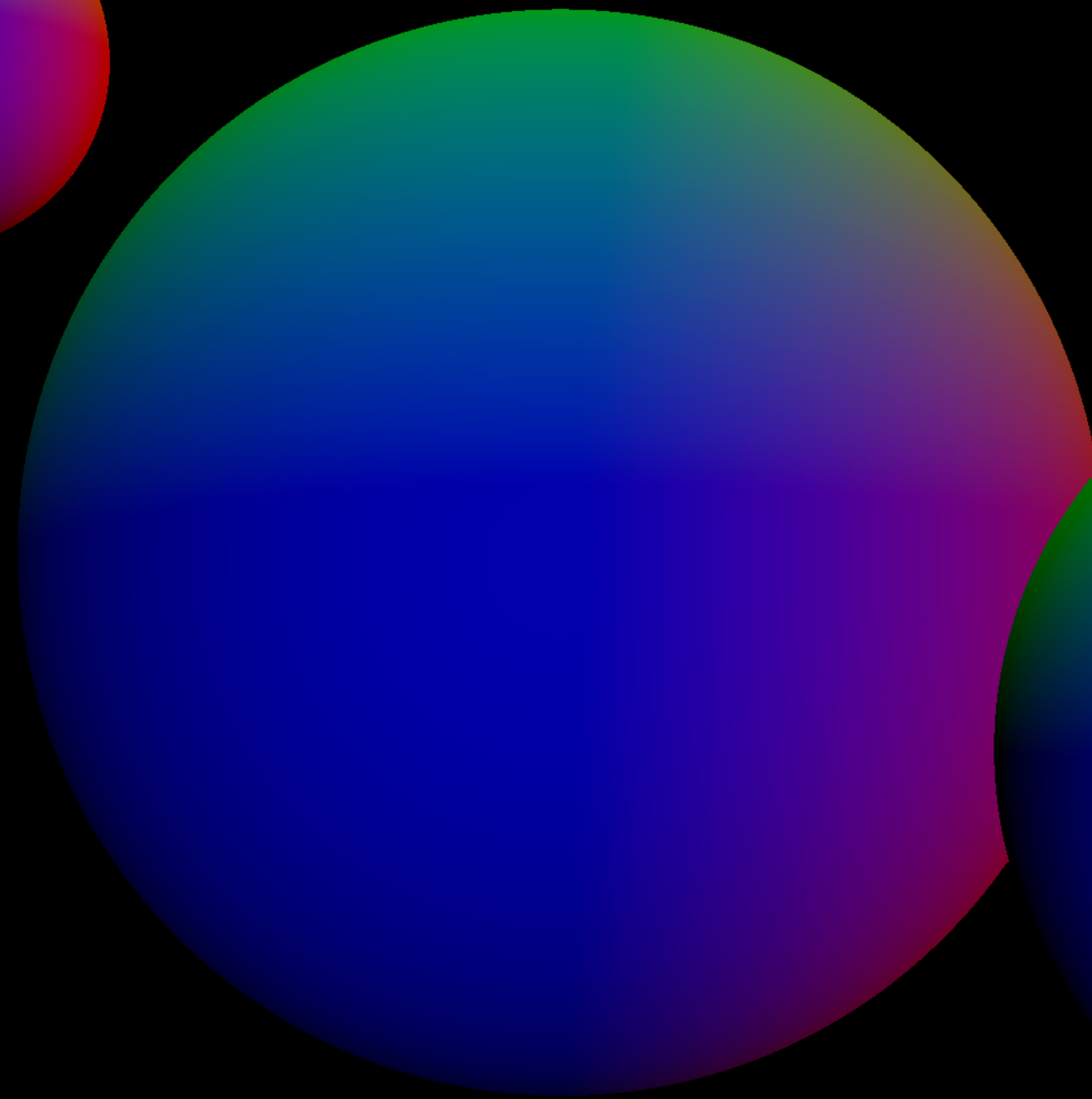
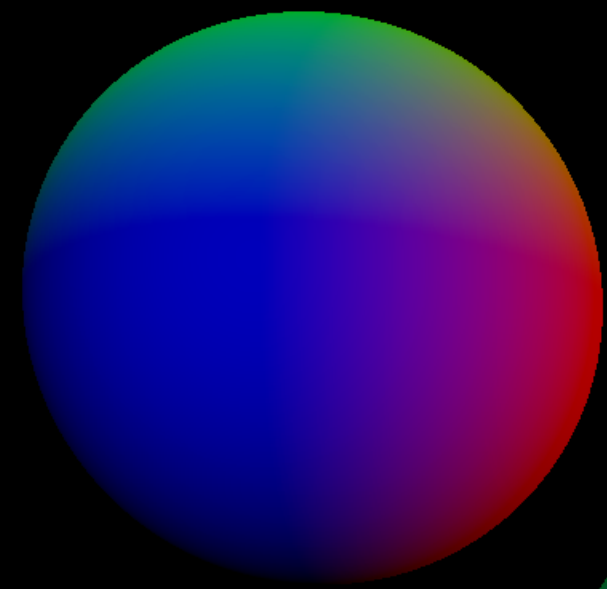




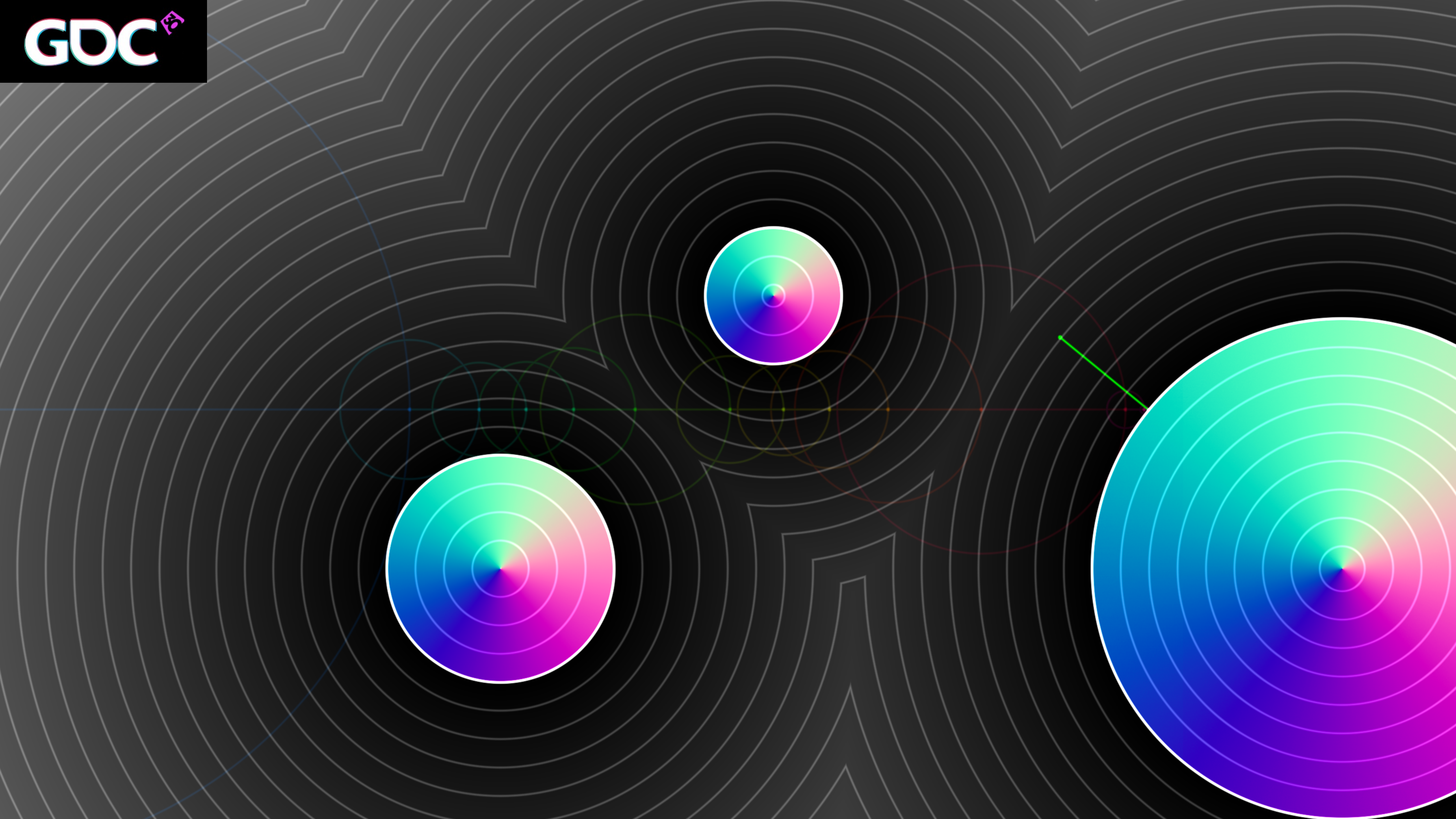




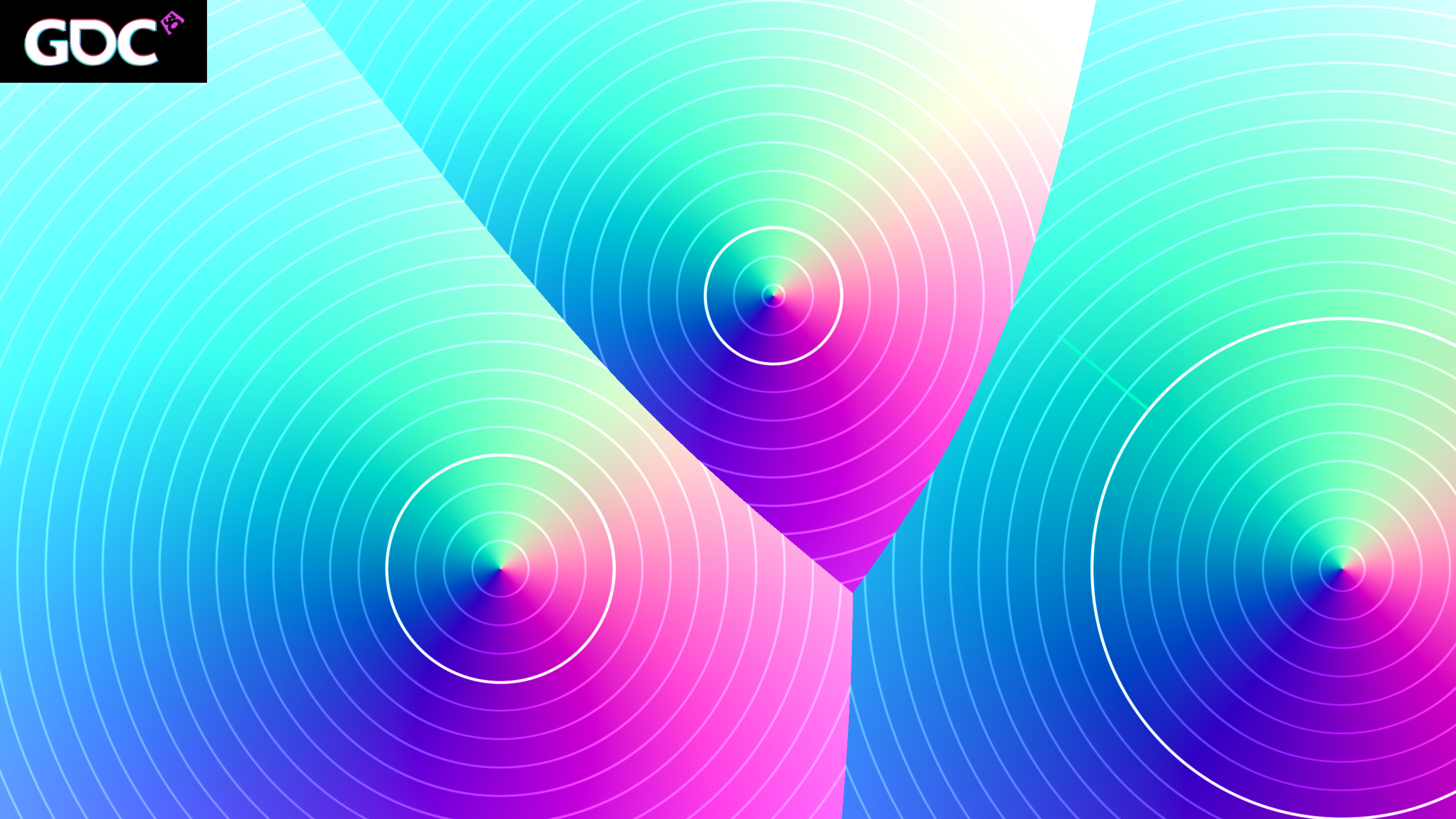




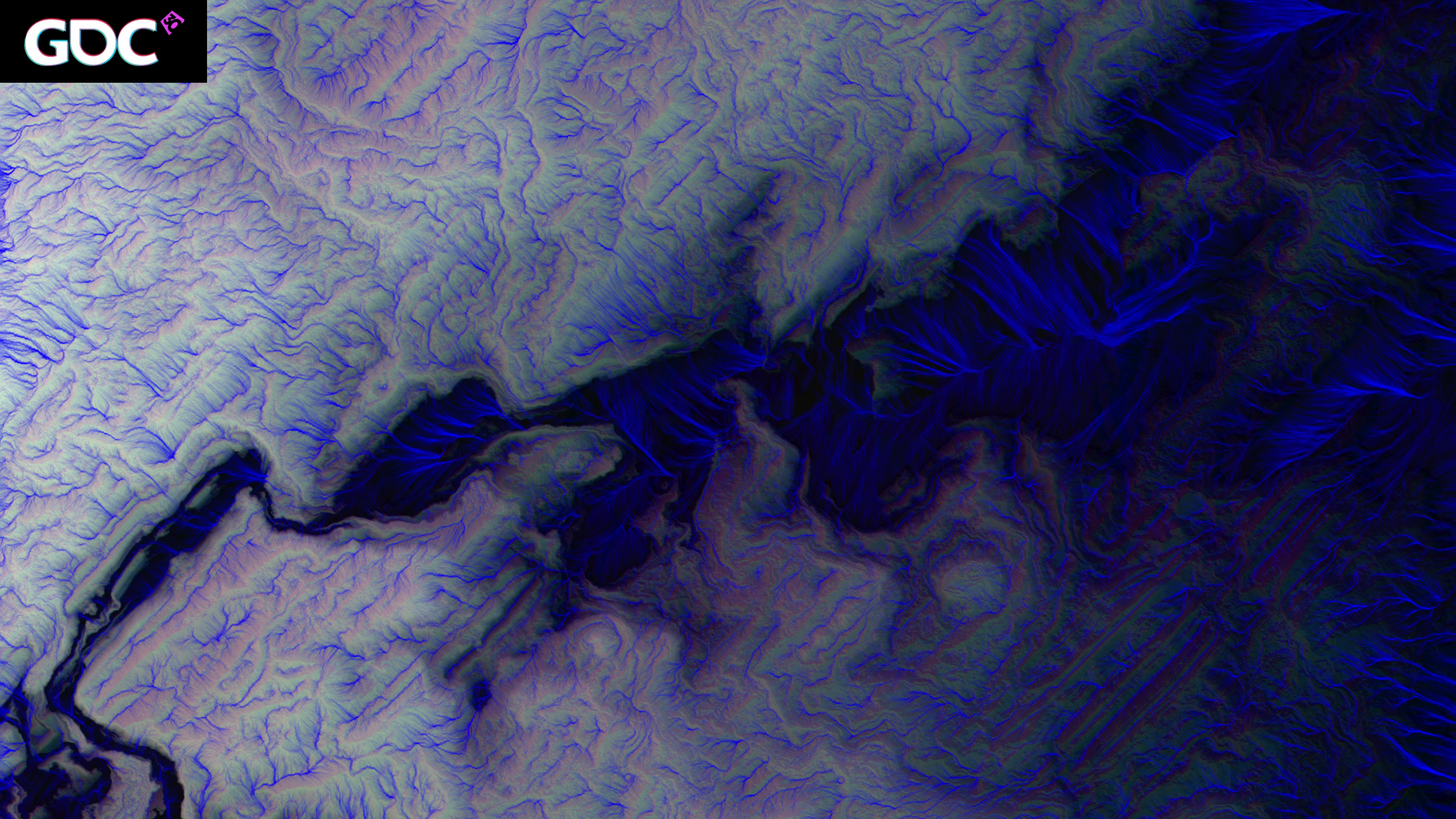




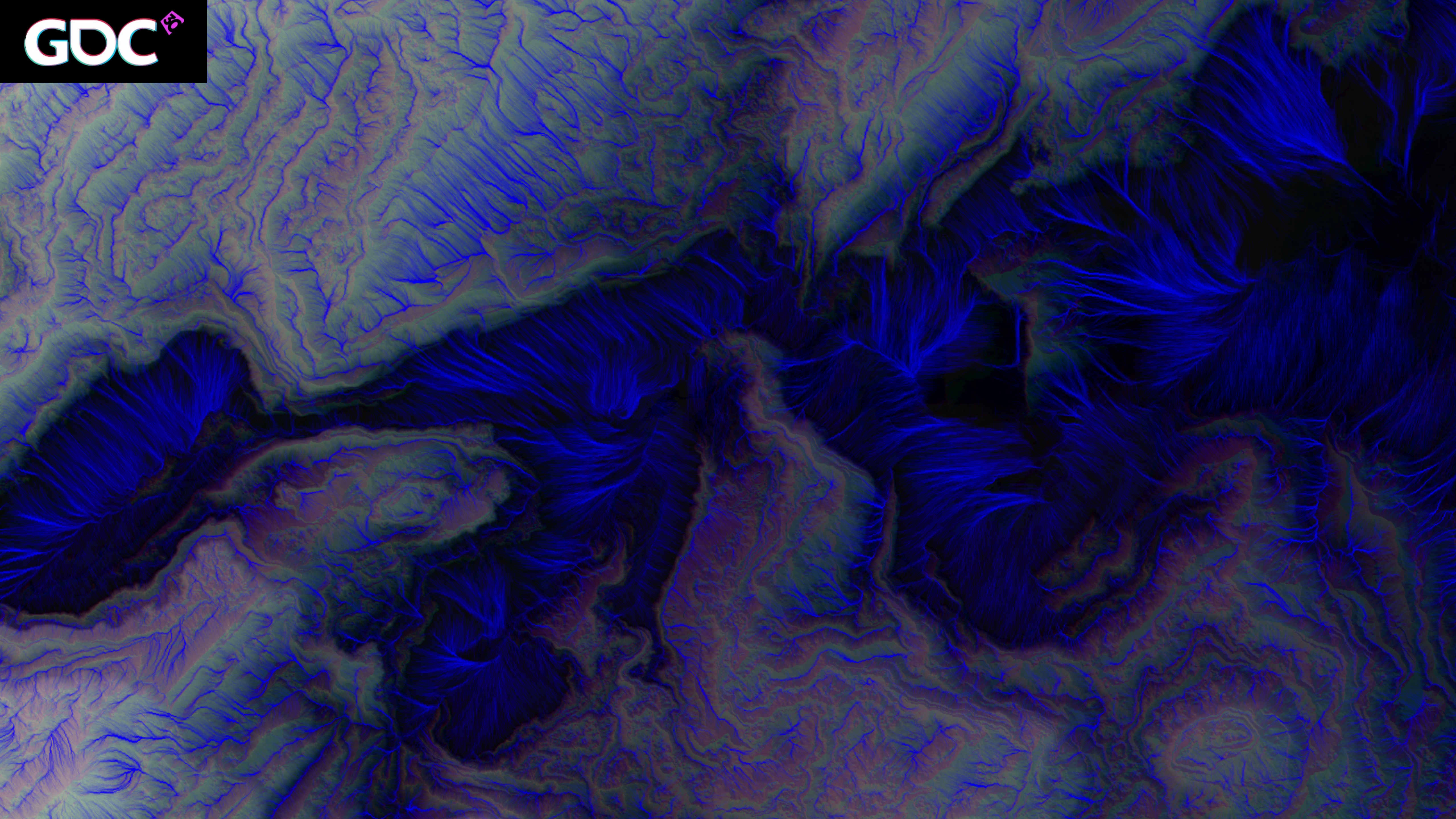




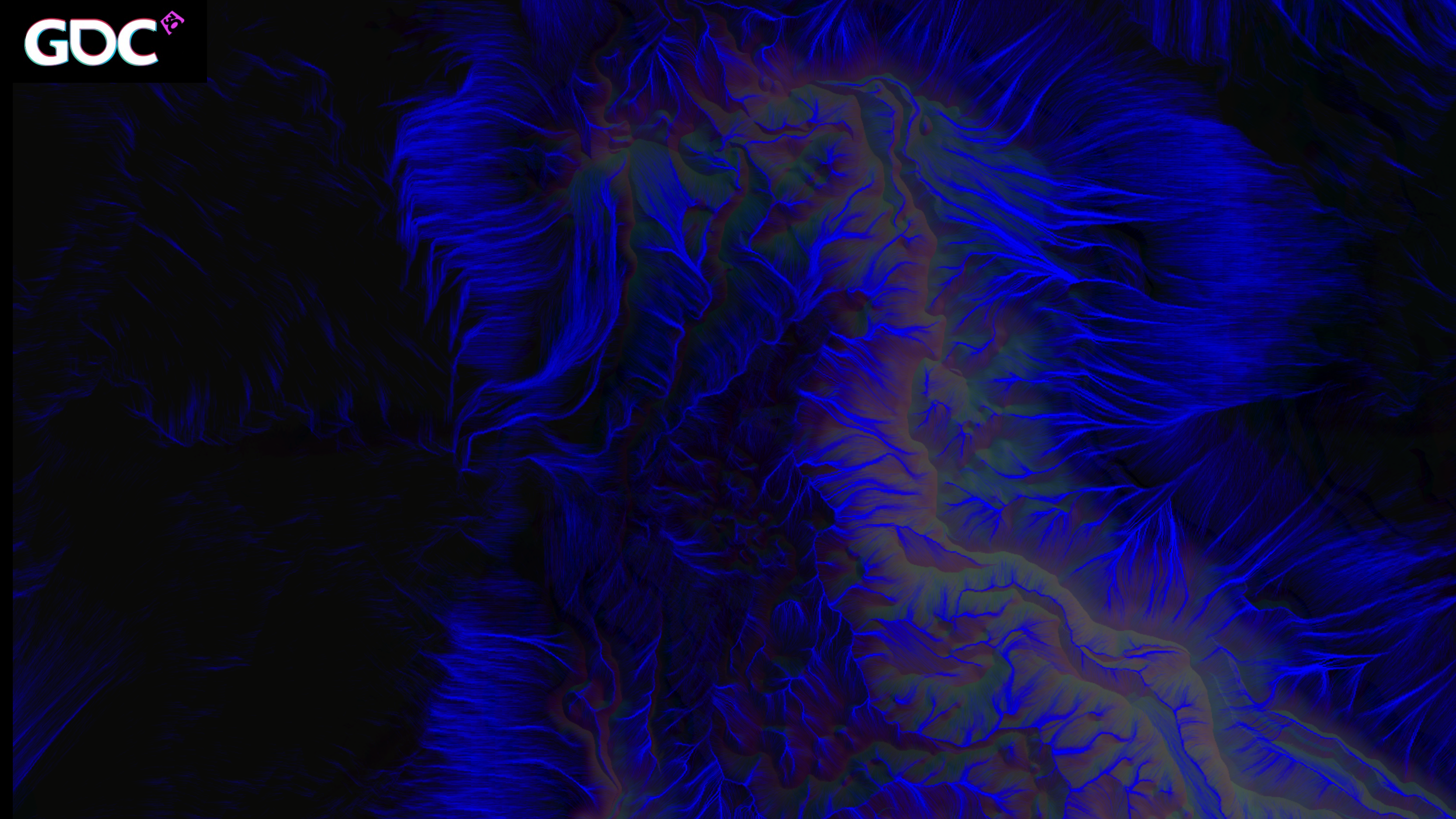




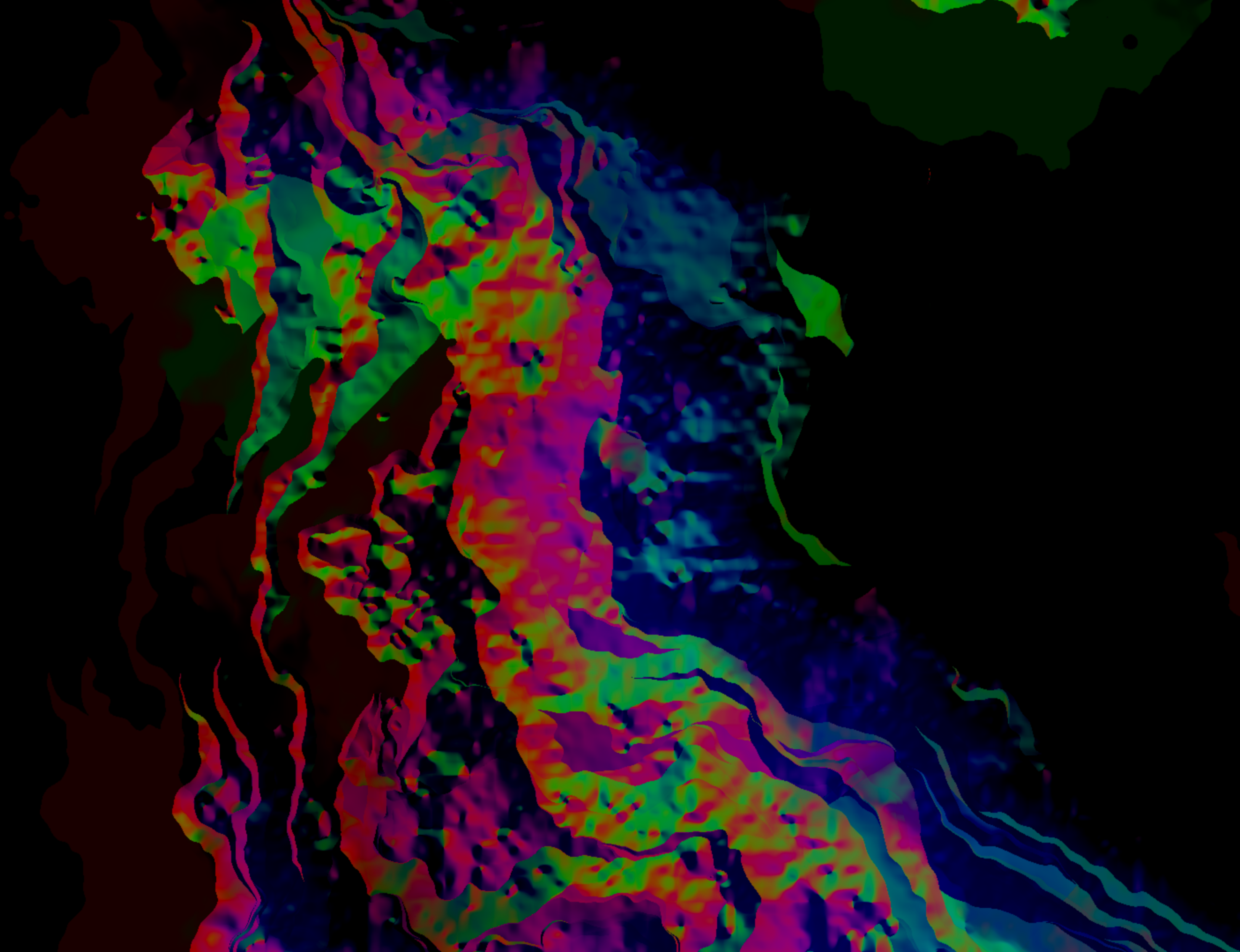


















# Lights





